Vacuum fluctuations and Casimir force

First lecture :

a short history of quantum fluctuations …

a brief introduction to modern quantum optics …

Second lecture :

the Casimir force between two flat mirrors at rest in vacuum

effects of imperfect reflection (finite conductivity)

comparison between theory and experiments

search for a violation of Newton force law at short distances

Third lecture :

geometry of the plates – effect of roughness

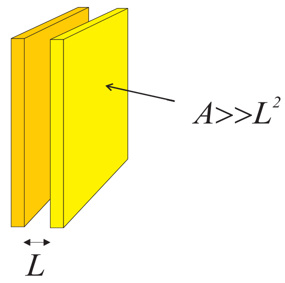
motion of the plates – “dynamical Casimir effect”

Tests of the Casimir force

- Casimir formula (1949)

$$F_{\text{Cas}} = -\frac{\hbar c \pi^2}{240\, L^4} A$$

$$E_{\text{Cas}} = -\frac{\hbar c \pi^2}{720\, L^3} A$$



- A lot of simplifications
  - plane parallel mirrors
  - perfect reflection
  - zero temperature
  - perfectly flat surfaces

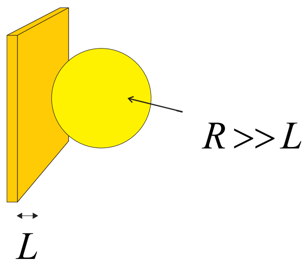
attraction measured as a pressure $P_{\text{Cas}} \equiv \frac{F_{\text{Cas}}}{A}$

$$P_{\text{Cas}} \approx -10^{-3}\,\text{Pa} \times \left(\frac{\mu\text{m}}{L}\right)^4$$

B. Duplantier in Poincaré Seminar on Vacuum Energy (2002)

The Casimir force in the plane-sphere geometry

- Recent experiments performed in the plane-sphere geometry
  - theory uses the proximity force approximation
  - contributions of the surface elements added up independently



- For the plane-sphere geometry (if *R>>L*)

$$F_{PS} = \int d^2x \frac{F_{PP}(x)}{A}$$

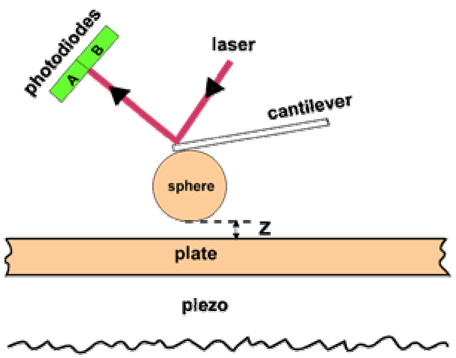
$$F_{PS} = 2\pi R \frac{E_{PP}}{A}$$

This is not a theorem : Casimir forces are not additive

Riverside experiment (Mohideen *et al*)

Atomic force microscope (AFM)

- Plane-sphere geometry
- Sphere (100µm) and plane covered with gold
- Distance 60-900nm
- Optical readout
- Experimental accuracy better than 2% at the smallest separations



Courtesy U. Mohideen

B.W. Harris *et al* Phys. Rev. A62 052109 (2003)

Riverside results (Mohideen *et al*)

Correct agreement with theory …

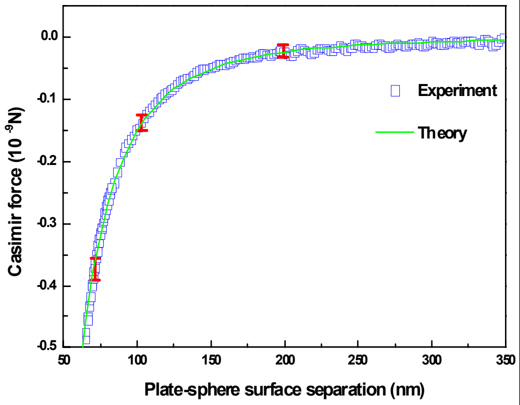
… after having accounted for

large corrections

- Plane-sphere geometry
- Imperfect reflection

and small corrections

- Room temperature
- Surface roughness



Courtesy U. Mohideen

A. Lambrecht & S. Reynaud in Poincaré Seminar (2002), quant-ph/0302073

Why the tests are going on

- Casimir force is an important prediction of quantum theory which has been recently measured with a good precision
  - a force induced by vacuum fluctuations which is directly observable in the macroscopic world !
  - accurate comparison with experiments allows for a test of theoretical predictions
  - theory must take into account the differences between the ideal Casimir configuration and real experiments
- One of the few experimental ways for
  - approaching the puzzle of vacuum energy
  - searching for hypothetical new forces at short distances

Tests of the Newton force law

Yukawa correction to the Newton law

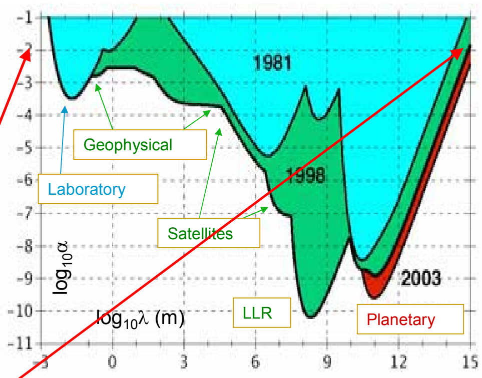
$$V(r) = -\frac{GMm}{r}\left(1 + \alpha e^{-\frac{r}{\lambda}}\right)$$

Windows remain open for deviations at short ranges

$\lambda < 1\,\mathrm{mm}$

or long ranges

$\lambda > 10^{16}\,\mathrm{m}$



Courtesy : J. Coy, E. Fischbach, R. Hellings, C. Talmadge, and E. M. Standish (2003)

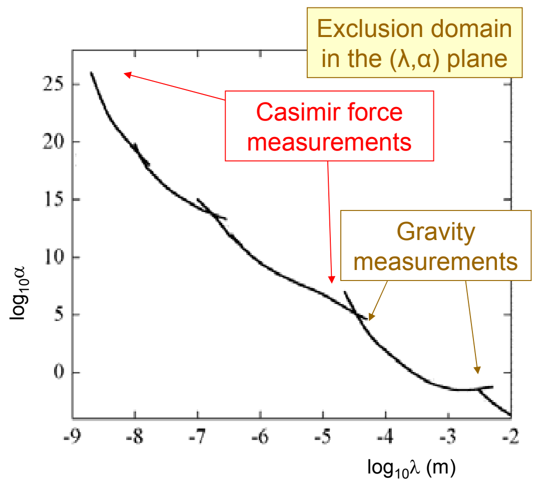
The Search for Non-Newtonian Gravity, E. Fischbach & C. Talmadge (1998)

Short range tests of the Newton law

- Gravity measurements at short distances

  λ > a few 10µm

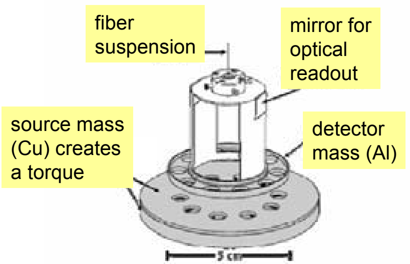
- At even shorter distances tests of Casimir force



E. Adelberger *et al* Annu. Rev. Nucl. Part. Sci. (2003) *hep-ph/0307284*

Adelberger *et al* (U. Washington)

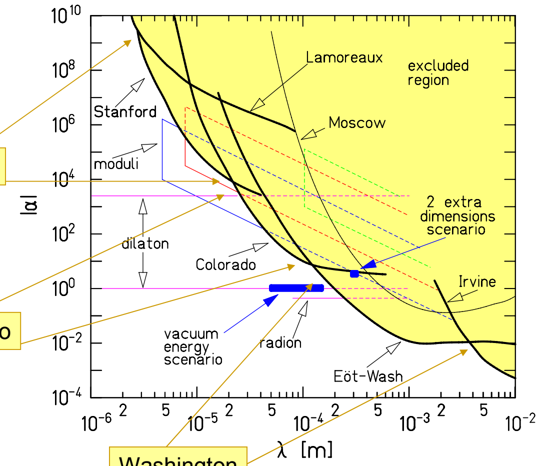
- Eöt-Wash : "Missing-mass" torsion balance
  Two disks with holes ; the attractor is rotated uniformly
  Separations down to 200μm : the best limits for 200μm<λ<3mm



The Newtonian signal is largely, but not completely, cancelled

C.D. Hoyle *et al* Phys. Rev. Lett. 86, 1418 (2001)

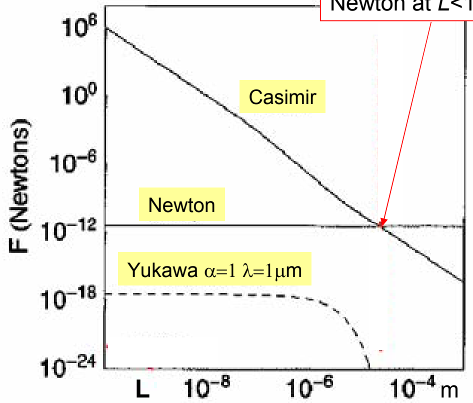
Short distance gravity tests : Summary of the results



E. Adelberger *et al* Annu. Rev. Nucl. Part. Sci. (2003) *hep-ph/0307284*

Newton vs Casimir : the challenge

For two Cu plates (1cm x 1cm x 1mm), Casimir dominates Newton at *L*<10μm



As an hypothetical new force would appear as a difference between experimental and theoretical values, these two values have to be obtained independently with the same care and accuracy

Coulomb force must also be controlled

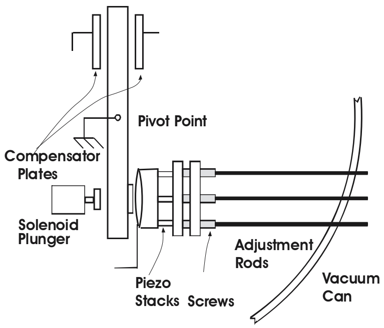
Recent Casimir force measurements

- Lamoreaux 1997 : Torsion pendulum with electrostatic compensation;
  Plane & sphere *R*=12.5cm at *L*=0.6-6μm; Accuracy ~5% (??)
- Mohideen *et al* 1998-... : Atomic force microscope (AFM)
  Plane & sphere *R*=100μm at *L*=100-500nm; Accuracy 1-2%
- Ederth 2000 : Crossed cylinders
  R=10mm at *L*=20-100nm
- Capasso et al 2001 : Micro-Electro-Mechanical Systems
  Plane & sphere R=100μm at L=100-500nm
- Onofrio *et al* 2002 : Casimir geometry
  Two parallel planes at *L*=0.5-3μm; Accuracy ~15%
- Decca *et al* 2003-... : Measurement between dissimilar metals
  Plane & sphere *R*=600μm at *L*=200-1200nm; Accuracy 1-2%

Lamoreaux (Seattle)

The first “modern” experiment

- Torsion pendulum with large electrostatic force used for compensation
- Correct agreement at shortest distances
- Accuracy announced at ~5% but not mastered
- Temperature effect not seen at largest distances (up to 6µm)



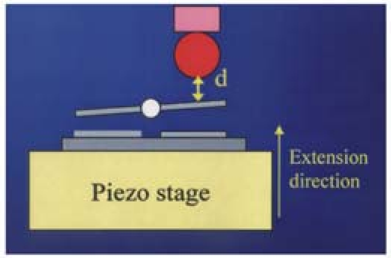
Courtesy S. Lamoreaux

S. Lamoreaux Phys. Rev. Lett. 78 (1997)

Capasso *et al* (Bell Labs)

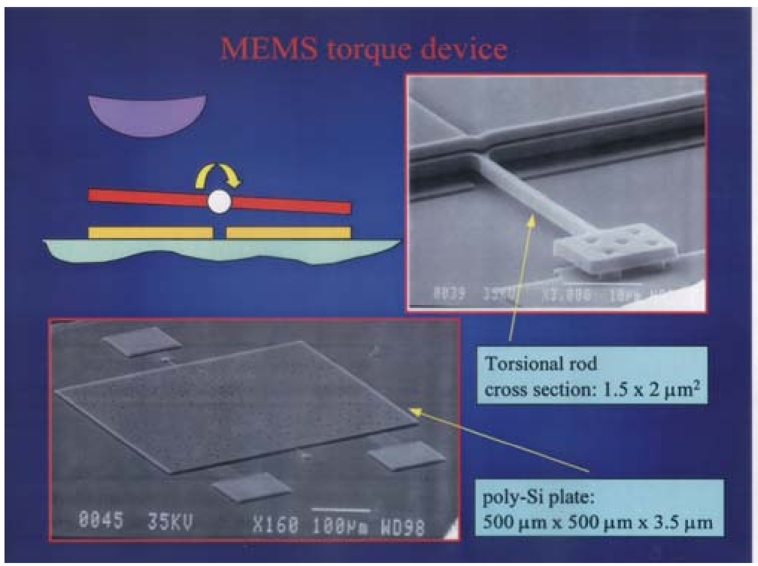
- Micro-electro-mechanical systems (MEMS)
- Poly-silicon plate hold by a torsional rod

- Sphere (100µm) and plane covered with gold
- Distance 100-500nm
- Capacitive readout



Courtesy F. Capasso

H.B. Chan *et al* Science 291, 1941 (2001), PRL 87, 211801 (2001)



Courtesy F. Capasso

Fischbach *et al* (Purdue)

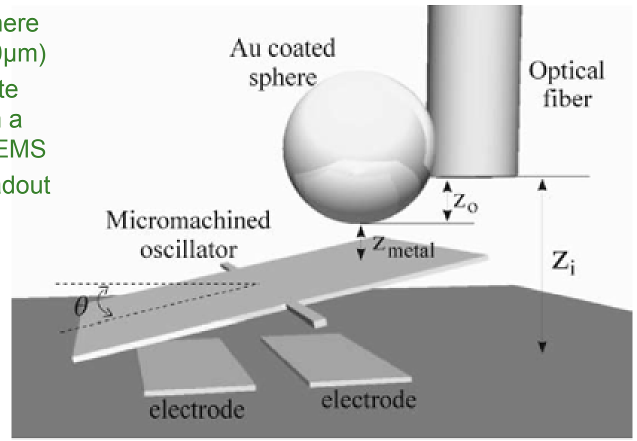
Au-coated sphere (R=100-600µm)

Cu-coated plate mounted on a torsional MEMS

Capacitive readout

Static or dynamic measurements

L = 260-1200nm

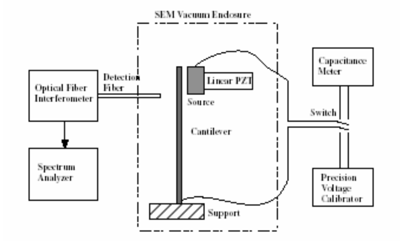


R. Decca *et al* Phys. Rev. D68 (2003)

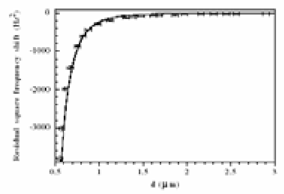
Bressi *et al* (Padova)

The only recent experiment in the Casimir geometry

- Two parallel planes at 0.5-3µm
- Dynamical measurement (shift of the vibration frequency of the flexible plate)
- Readout through an optical fiber interferometer
- Accuracy ~15%

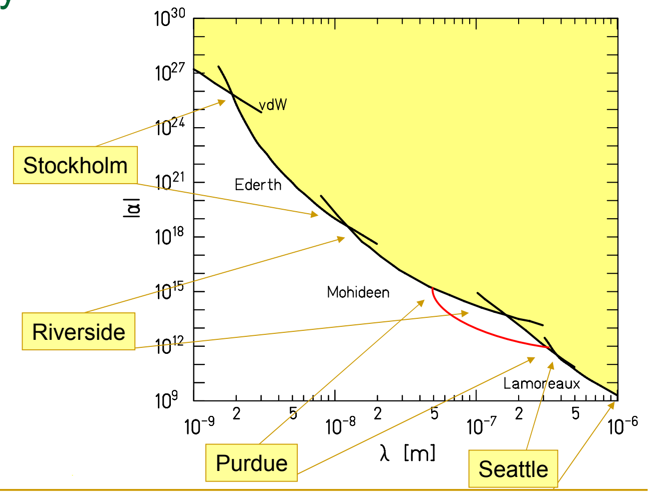


Courtesy R. Onofrio



Bressi, Carugno, Onofrio, Ruoso PRL 88 (2002)

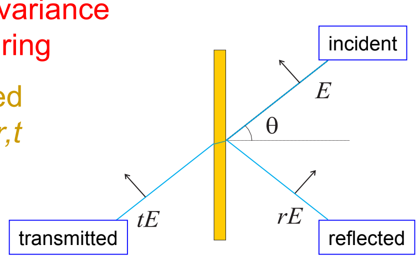
Casimir tests of the Newton law : Summary of the results



E. Adelberger *et al* Annu. Rev. Nucl. Part. Sci. (2003) *hep-ph/0307284*

Imperfectly reflecting mirrors

- Perfectly plane, parallel and flat (but not perfectly reflecting) mirrors obey lateral translation invariance and show specular scattering
- Such mirrors are described by scattering amplitudes *r,t* which depend on
  - frequency ω
  - incidence angle θ
  - polarization *p=TE,TM*



The Casimir force can be expressed in terms of the reflection amplitudes

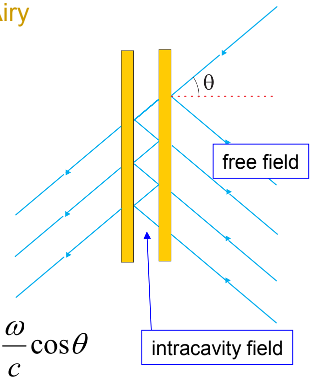
Jaekel, Reynaud, J. Physique (1991), *arXiv:quant-ph/0101067*

Cavity with imperfectly reflecting mirrors

- The spectral density is multiplied by the Airy function for the intracavity field

$$g = \frac{1 - \left| r_1 r_2 e^{2ik_z L} \right|^2}{\left| 1 - r_1 r_2 e^{2ik_z L} \right|^2}$$

- r_1 and r_2 are the reflection amplitudes seen by the intracavity field
- k_z is the longitudinal wave-vector $k_z = \frac{\omega}{c}\cos\theta$



This is a theorem which has been proven for lossy as well as lossless mirrors

C. Genet, A. Lambrecht & S. Reynaud, Phys. Rev. A67 (2003)

Radiation pressure on the mirrors

- For each field mode, vacuum radiation pressures differ
  - on the outer side $\left(\frac{\hbar\omega}{2}+\bar{n}\hbar\omega\right)\cos^2\theta$
  - and
  - on the inner side $\left(\frac{\hbar\omega}{2}+\bar{n}\hbar\omega\right)\cos^2\theta\times g$

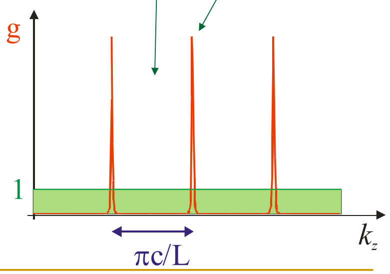
attractive contributions

repulsive contributions

- The net pressure is proportional to

$$g-1=f+f^*$$

$$f=\frac{r_1r_2e^{2ik_zL}}{1-r_1r_2e^{2ik_zL}}$$

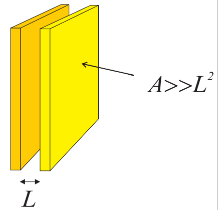


Fluctuations-dissipation theorem has been used

Sum over the modes

- The force is an integral over all field modes

$$F_{\mathrm{PP}}=\sum_{\mathrm{modes}}\left(\frac{\hbar\omega}{2}+\bar{n}\hbar\omega\right)\cos^2\theta\ (1-g)$$

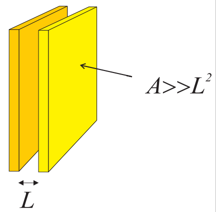


  - The sum over the modes includes evanescent waves as well as ordinary propagating waves
- This formula is valid for arbitrary mirrors
  - the reflection amplitudes are still to be specified through measurements or modeling of mirrors
  - we have only assumed specular reflection

Long-range force: electronic influence between the mirrors negligible

Final expression of the Casimir force …

- Using causality properties
  - the formula can also be written as an integral over imaginary frequencies



- The final scattering formula is regular
  - no need for further regularization

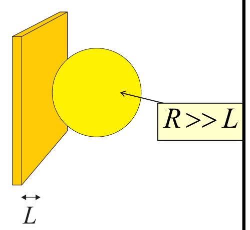
    for any causal mirrors
- It goes to the Casimir formula for $r_1\ r_2\to 1$ and $T\to 0$

$$F_{\mathrm{Cas}}=-\frac{\hbar c\pi^2}{240\,L^4}A$$

C. Genet, A. Lambrecht & S. Reynaud, Phys. Rev. A67 (2003)

… in the plane-sphere geometry

- At zero temperature
- in the plane-sphere geometry



$$F_{\mathrm{PS}}=2\pi R\frac{E_{\mathrm{PP}}(L)}{A}$$

$$E_{\mathrm{PP}}(L)=\int_L^\infty \mathrm{d}L'\ \mathrm{F}_{\mathrm{PP}}(L')$$

- These expressions represent the QED prediction to be compared with measurements
  - reflection amplitudes still to be specified

Proximity force approximation used for the plane-sphere geometry

Models of “real” mirrors

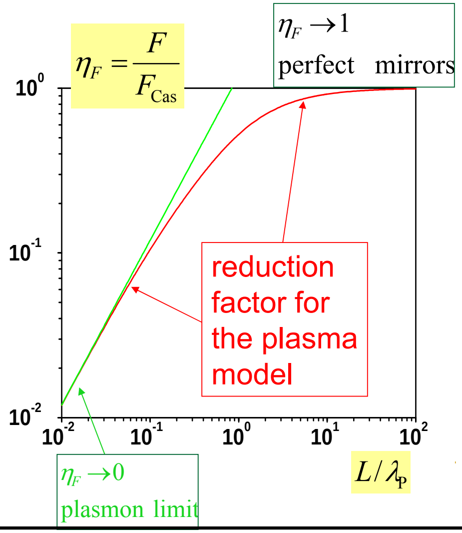
- Simplest model for metallic mirrors
  - Fresnel reflection laws
  - plasma model for the gas of conduction electrons

$$\varepsilon(\omega) = 1 - \frac{\omega_P^2}{\omega^2}$$

  - Plasma frequency and wavelength

$$\omega_P = \frac{2\pi c}{\lambda_P}$$

- Reduction of the force calculated with respect to the ideal Casimir formula



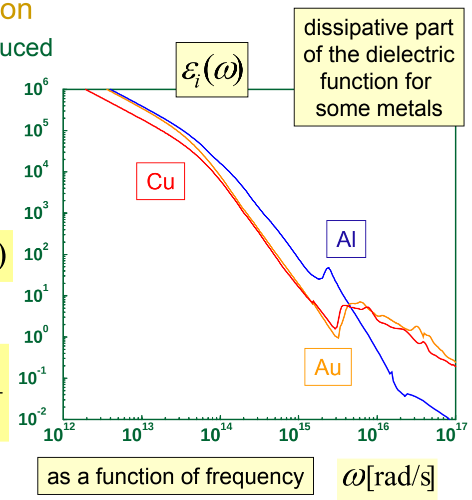
More complete description of metals

- Plane, parallel and flat mirrors showing specular reflection
  - Scattering amplitudes deduced from Fresnel laws
  - Optical response of electrons described by tabulated dielectric function

$$\varepsilon(\omega) = \varepsilon_r(\omega) + i\varepsilon_i(\omega)$$

and causality relations

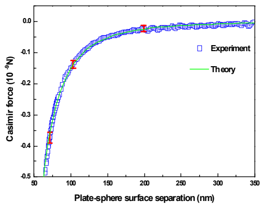
$$\varepsilon_r(\omega) = 1 + \frac{2}{\pi}\mathrm{P}\int_0^\infty \mathrm{d}x \frac{x\varepsilon_i(x)}{x^2 - \omega^2}$$

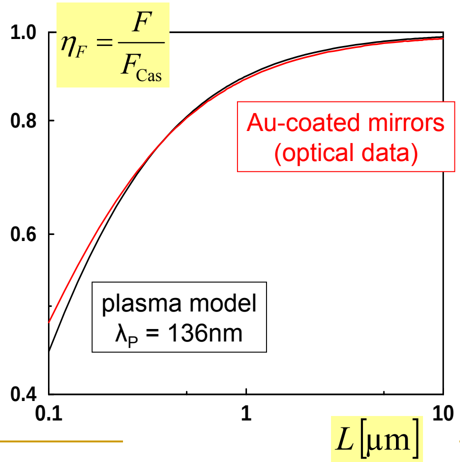


Results for two Au-covered mirrors

global behaviour well reproduced by the plasma model

integration of tabulated optical data needed for obtaining accurate predictions





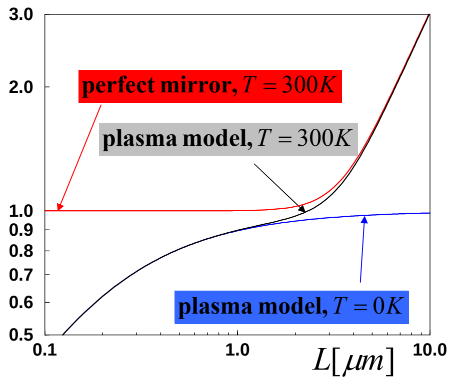
A. Lambrecht & S. Reynaud, Eur. Phys. J. D8 309 (2000)

Finite temperature correction

- Radiation pressure of thermal fluctuations has to be added to that of vacuum fluctuations

$$\frac{\hbar\omega}{2} \to \frac{\hbar\omega}{2} + \frac{\hbar\omega}{e^{\hbar\omega/k_B T} - 1}$$

  - at T=300K, important corrections for L>3µm
  - temperature & plasma corrections are correlated at intermediate distances L≈1-3µm



C. Genet, A. Lambrecht & S. Reynaud, Phys. Rev. A62 (2000)

Conclusions (at the moment …)

- The Casimir force is now measured with a good experimental accuracy ~ 1-2%
- Theory and experiment agree at the same level in the distance range 100nm < L < 500nm
- The effect of imperfect reflection is precisely measured and accurately calculated
- Theoretical discussions are still extremely active about the effect of non zero temperature (for dissipative mirrors)
- And the effect of thermal fluctuations has still to be detected unambiguously in experiments

Casimir and Casimir-Polder

- Variation of the reduction factor for the energy as a function of distance
  - at large distances, limit of perfect mirrors

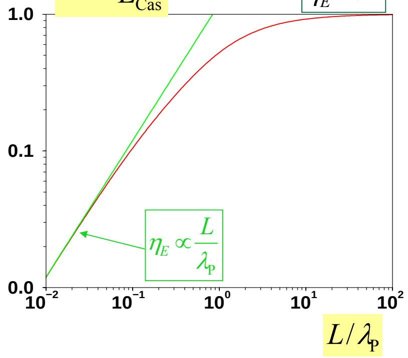
$$E_{\mathrm{Cas}} = -\frac{\hbar c \pi^2 A}{720}\frac{1}{L^3}$$

  - at short distances, different scaling law

$$E = -\frac{\alpha \hbar c \pi^2 A}{480\,\lambda_{\mathrm{P}}}\frac{1}{L^2} \qquad \alpha \approx 1.193$$

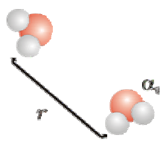
$$\eta_E = \frac{E}{E_{\mathrm{Cas}}}$$

$\eta_E \to 1$



This is reminiscent of the Casimir-Polder law for atomic forces

Casimir-Polder crossover



- From the Van der Waals interaction between neutral atoms in their ground states

Instantaneous interaction for $r \ll \lambda_A$

$$E \propto -\frac{\hbar c \pi \alpha^2}{\lambda_A r^6}$$

London 1930

Retarded interaction for $r \gg \lambda_A$

$$E_{\mathrm{C\text{-}P}} = -\frac{23\hbar c \alpha_0^2}{4\pi r^7}$$

Casimir-Polder 1948

- to the Casimir interaction between two mirrors

Instantaneous interaction for $L \ll \lambda_p$

$$E \propto -\frac{\hbar c \pi^2 A}{480 \lambda_{\mathrm{P}} L^2}$$

Lifshitz 1956

Retarded interaction for $L \gg \lambda_p$

$$E_{\mathrm{Cas}} = -\frac{\hbar c \pi^2 A}{720 L^3}$$

Casimir 1949

Casimir force and surface plasmons

- Surface plasmons ≡ collective electron excitations coupled to evanescent fields and propagating on the interface between each metallic bulk and vacuum
  - For each metallic bulk, dispersion relation versus transverse wavevector k

$$\omega_{\mathrm{sp}}^2 = \frac{\omega_{\mathrm{P}}^2 + 2c^2\mathrm{k}^2 - \sqrt{\omega_{\mathrm{P}}^4 + 4c^4\mathrm{k}^4}}{2}$$

  - Plasmons on the two bulks are coupled by Coulomb law

$$\omega_{\pm}^2 \approx \frac{\omega_{\mathrm{P}}^2 (1 \mp e^{-\mathrm{k}L})}{2}$$

- At short distances, the Casimir force can be seen as the effect of Coulomb interaction between surface plasmons

$$E_{sp} = A \int \frac{d^2\mathbf{k}}{(2\pi)^2}\left(\frac{\hbar\omega_+}{2} + \frac{\hbar\omega_-}{2} - 2\frac{\hbar\omega_{sp}}{2}\right) = -\frac{\hbar c \alpha \pi^2 A}{480 \lambda_p L^2}$$

see also F. Intravaia & A. Lambrecht, Phys. Rev. Lett. (2005)