Vacuum fluctuations and Casimir force

First lecture :

from quantum fluctuations to modern quantum optics

Second lecture :

the Casimir force between two flat motionless mirrors

invariance vs time and lateral space translations

Third lecture :

perturbations breaking these symmetries

geometry and/or roughness of the plates

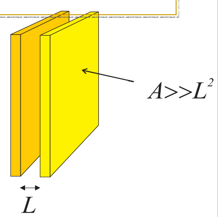
motion of the plates – “fluctuations-dissipation” in vacuum – “dynamical Casimir effect” – generation of photons from motion in vacuum

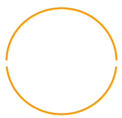
A few references

- Reflection on moving mirrors as an analogy to gravitational perturbations of quantum vacuum
  - B.S. de Witt, Phys. Rep. 19 (1975) 295
- Radiation from a perfect mirror moving in vacuum
  - S.A. Fulling & P.C.W. Davies, Proc. R. Soc. A348 (1976) 393
- Generation of photons inside a cavity built up with two perfect mirrors moving in vacuum
  - G.T. Moore, J. Math. Phys. 11 (1970) 2879
- More references in
  - M.-T. Jaekel & S. Reynaud, Rep. Progr. Phys. 60 (1997) 863 = arXiv:quant-ph/9706035

Casimir force and geometry

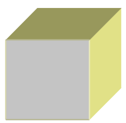
- Attraction between two plane plates
  - for perfect reflectors as well as dielectric mirrors
  - change of sign requires a magnetic mirror





- Repulsion for a sphere
  - T. Boyer Phys. Rev. 1968

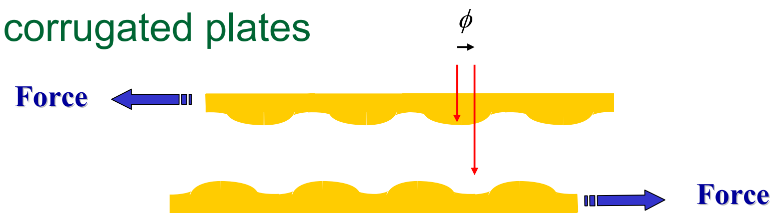
- Attraction or repulsion for a parallelepiped
  - depending on the ratio of the sides



- Sensitive dependence of Casimir force to boundary geometry
  - but no experimental result except for corrugation

R. Balian & B. Duplantier arXiv:quant-ph/0408124 (2004)

Lateral Casimir force between two corrugated plates

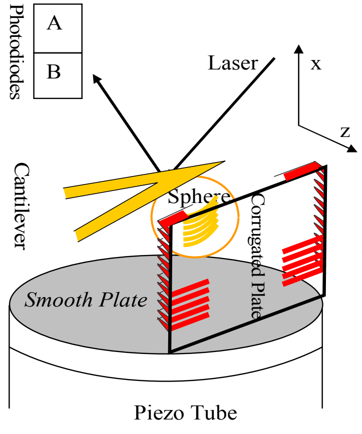


- Break Right-Left Translational Symmetry
  - Lateral forces
- For long corrugation wavelength, this force can be calculated within the Proximity Force Approximation

$$\left\langle \frac{1}{\left(L + a\sin kx - a\sin(kx-\phi)\right)^3} \right\rangle = \frac{1}{L^3}\left(1 + \frac{12\,a^2}{L^2}\cos^2\frac{\phi}{2} + \ldots\right)$$

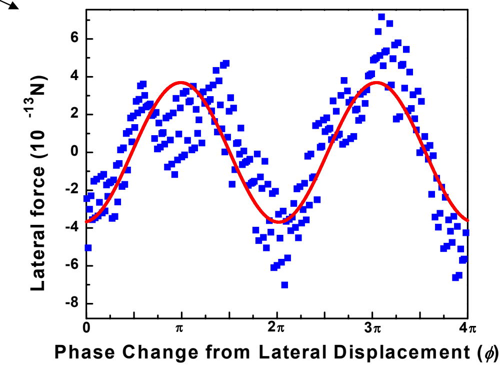
R. Golestanian & M. Kardar PRL (1997), PRA (1998)

Experimental demonstration



Courtesy U. Mohideen

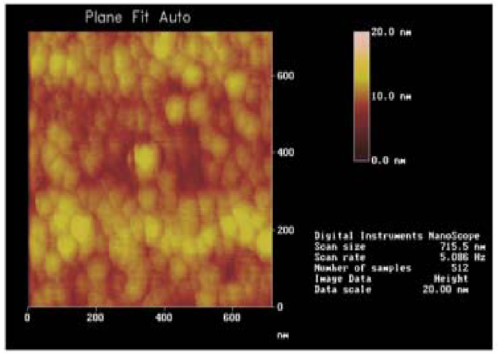
- Performed on the Riverside Casimir experiment on two surfaces separated by 221nm



F. Chen, U. Mohideen, G. Klimchitskaya & V. Mostepanenko PRL 88 (2002)

Normal Casimir force and roughness

- Real plates show a rough surface
- Roughness correction to the normal Casimir force usually calculated within PFA



Courtesy U. Mohideen

$$\langle E\left(L+h_1(x,y)-h_2(x,y)\right)\rangle = E(L)+\frac{d^2E}{2\,dL^2}\left\langle (h_1-h_2)^2\right\rangle$$

- Small correction for roughness amplitudes ~1nm (fortunately)

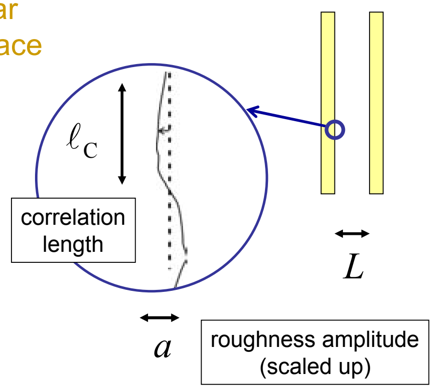
Inaccuracy of Proximity Force Approximation

- PFA disregards non-specular scattering on the rough surface
- PFA can only be valid for nearly flat surface

$\ell_C >> L, \lambda_P$

- Real plates do not obey this condition

$\ell_C \approx 10-500\text{nm}$ $\quad \lambda_P \approx 136\text{nm}$ $\quad L \approx 50-1000\text{nm}$

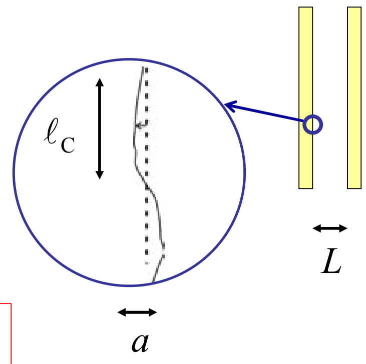


C. Genet, A. Lambrecht, P. Maia Neto & S. Reynaud, Europhys. Lett. (2003)

Beyond the Proximity Force Approximation

- Non specular scattering on a rough surface :
  - the surface is static → no frequency change $\Delta\omega = 0$
  - the surface is not flat → lateral wavevector is changed $\Delta k \approx \ell_C^{-1}$
- The effect can be evaluated analytically as a perturbation in the roughness amplitude

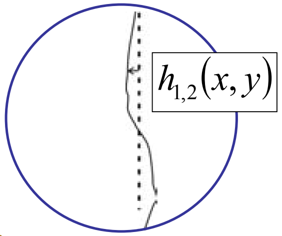
$a \approx 1\text{nm} \quad \rightarrow \quad a << \ell_C, L, \lambda_P$



P. Maia Neto, A. Lambrecht & S. Reynaud, Phys. Rev. A72 (2005)

Simplifying assumptions

- Spatial average replaced by a statistical average $A >> \ell_C^2$ $\quad RL >> \ell_C^2$
  - Profiles *i=1,2* described by roughness spectra



$$\sigma_{ij}[\mathbf{k}] = \int dA \exp(-i\mathbf{k}.\mathbf{r}) < h_i(\mathbf{r}) h_j(\mathbf{0}) >$$

- Surfaces uncorrelated $\quad \sigma_{12}[\mathbf{k}] = 0$
- Mirrors described by the plasma model with the same λ_P

- Analytical expression (in a perturbation in *h*)

$$\delta E^{\text{rough}} = \int \frac{d^2k}{4\pi^2} G[k]\sigma[k]$$

$$\sigma[k] = \sigma_{11}[k] + \sigma_{22}[k]$$

Corrugation not treated here

Spectral sensitivity to roughness

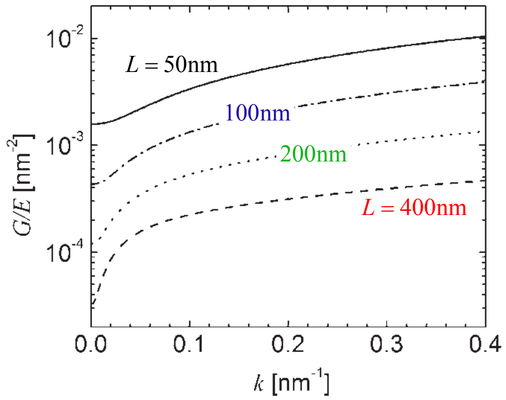
- *G*(k) obtained as an integral over the scattering amplitudes mixing different wavevectors and polarizations

PFA

$$G[0] = \frac{1}{2}\frac{d^2E}{dL^2}$$

- Proximity force approximation recovered at the limit k→0

- For arbitrary k, the correction is always larger than in PFA



Departure from PFA

- Departure measured by a dimensionless factor which depends on the wavevector k (or on the scale k^{-1})

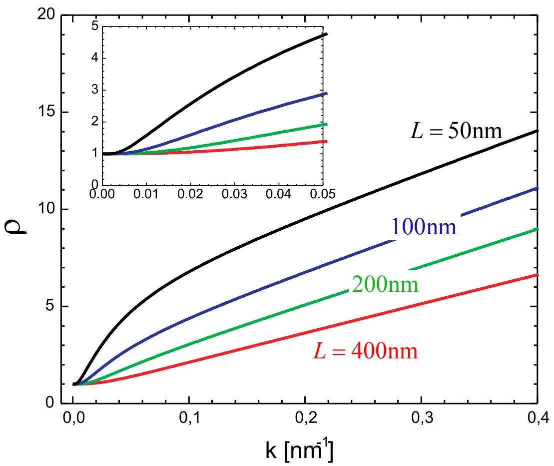
$$\rho[k] = \frac{G[k]}{G[0]} \geq 1$$

- Large scale limit

$$\rho[k] = 1$$
$$\lambda_P, L << k^{-1}$$

- Small scale limit

$$\rho[k] = \alpha k$$
$$k^{-1} << \lambda_P, L$$



A few limiting cases

- Small scale roughness and short distance

$$\rho = 0.4492\, kL$$
$$k^{-1} << L << \lambda_P$$

Small scale limit of the plasmon regime ; Maradudin & Mazur Phys. Rev. B (1980, 1981) (after a correction by a factor 2)

- Small scale roughness and long distances

$$\rho = \frac{14}{30\pi} k\lambda_P \quad ; \quad k^{-1} << \lambda_P << L$$

- Limit of perfect mirrors

$$\lambda_P << k^{-1}, L$$

Emig, Hanke, Golestanian, Kardar PRL (2001)

- In particular, perfect mirrors with small scale roughness

$$\rho[k] = \frac{1}{3} kL \quad ; \quad \lambda_P << k^{-1} << L$$

P. Maia Neto, A. Lambrecht & S. Reynaud, Europhys. Lett. (2005)

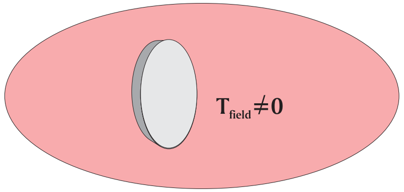
Motion in vacuum fluctuations

- Classical vacuum is absolutely empty
- But quantum vacuum is not empty, it is “filled” with field fluctuations !

- This raises a lot of questions
  - What are the effects of vacuum on moving mirrors ?
  - Are there observable in experiments ?
  - How do vacuum fluctuations preserve compatibility with the principle of relativity of motion ? …
- Some of these questions can be solved by considering vacuum as the limit at T=0 of a thermal equilibrium
  - Fluctuations-dissipation theorem applicable

M.-T. Jaekel & S. Reynaud, Rep. Progr. Phys. 60 (1997)

A mirror moving in a thermal field experiences a friction force (Einstein 1909-1917)



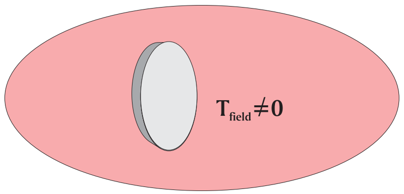
- Simple model
  - 2-dimensional spacetime
  - scalar field at $T \neq 0$
- For a mirror moving with a time-dependent position $q(t)$
  - the friction force is proportional to the velocity $q'(t)$

$$F_{diss}(t) = -\frac{\hbar \theta^2}{6\pi c^2} q'(t)$$

$$\theta = \frac{2\pi k_B T_{field}}{\hbar}$$

- No conflict with the principle of relativity of motion
  - a thermal field is not Lorentz invariant (it defines a specific frame)
  - this force disappears at $T=0$ (vacuum is Lorentz invariant)

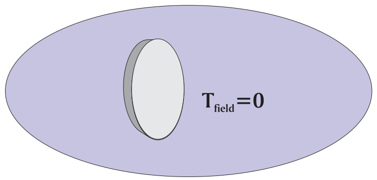
Fluctuations and dissipation in a thermal field



- For a mirror at rest
  - the mean force vanishes
  - radiation pressure of field fluctuations imply the presence of force fluctuations
- An exemple of (Einstein) fluctuations-dissipation relations
  - force fluctuations described by a noise spectrum $C[\omega]$
  - dissipation described by a linear susceptibility $\chi[\omega]$ in frequency domain
  - the two quantities are directly related through “linear response theory” (*ie* linear treatment of motion-induced perturbations to the static zeroth-order configuration)

$$F_{diss}[\omega] = \frac{\hbar \theta^2}{6\pi c^2} i\omega q[\omega]$$

Fluctuations and dissipation in vacuum



- For a mirror at rest
  - the mean force vanishes
  - radiation pressure of vacuum fluctuations imply the presence of force fluctuations
- For a mirror moving in vacuum
  - dissipation described by a linear susceptibility
  - the force appears only for a non uniform acceleration

$$F_{diss}[\omega] = \frac{\hbar}{6\pi c^2} i\omega^3 q[\omega]$$

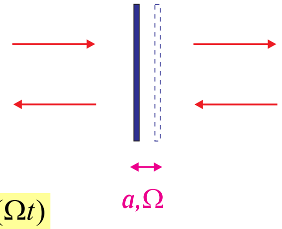
$$F_{diss}(t) = \frac{\hbar}{6\pi c^2} q'''(t)$$

- No conflict with Lorentz invariance of vacuum

Analogy with spontaneous emission for charges in vacuum

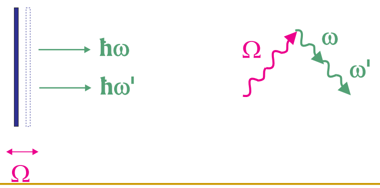
Experimental signatures

The dissipative force is exceedingly small
but the moving mirror emits radiation



- For an harmonic motion $q(t) = 2a\sin(\Omega t)$
  - number of emitted photons in a linearized approach $v << c$

$$N = \frac{\Omega T}{3\pi}\left(\frac{v}{c}\right)^2 \quad , \quad v = 2\Omega a$$



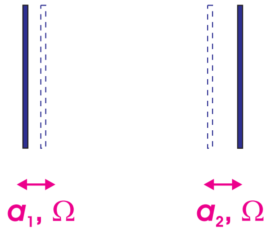
Radiation can be seen as a parametric emission induced by motion

➢ Too small to be observable in experiments

Resonant enhancement for a cavity

- Numbers largely improved for a cavity
  - a detectable number of photons can be expected for a high enough cavity finesse

$$F = \frac{\pi}{1 - r^2}$$



- For an harmonic motion
  - number of emitted photons in a linearized approach $v << c$

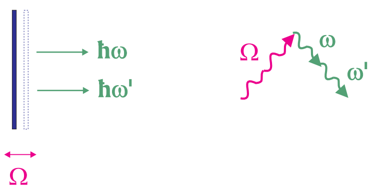
$$N = F\frac{\Omega T}{3\pi}\left(\frac{v}{c}\right)^2 \quad , \quad v = 2\Omega a$$

➢ The solution to be explored for an experimental demonstration

A. Lambrecht, M.-T. Jaekel & S. Reynaud, Phys. Rev. Lett. 77 (1996)

Even and odd modes

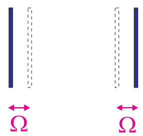
- Parametric excitation of cavity resonances by motion



$$\omega = n\frac{\pi}{\tau} \quad , \quad \tau = \frac{L}{c}$$

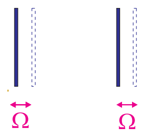
$$\Omega = \omega + \omega' = (n + n')\frac{\pi}{\tau} = K\frac{\pi}{\tau}$$

- Two sets of modes
  - breathing mode (K even)



$\Omega = \frac{2\pi}{\tau}, \frac{4\pi}{\tau}, \ldots \quad a_1 = -a_2$

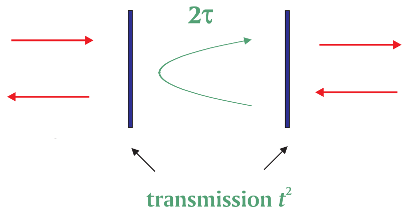
  - global oscillation (K odd)



$\Omega = \frac{3\pi}{\tau}, \frac{5\pi}{\tau}, \ldots \quad a_1 = a_2$

Photons inside and outside the cavity

- Each photon has a probability of $2t^2$ to escape from the cavity during one roundtrip time 2τ



- The number of photons radiated by the cavity (per unit of time)

$$\frac{N}{T} = \frac{\Omega}{3\pi}\left(\frac{v}{c}\right)^2 \frac{\pi / F}{(\pi / F)^2 + (\Omega\tau - K\pi)^2}$$

is directly related to the stationary number of photons inside the cavity

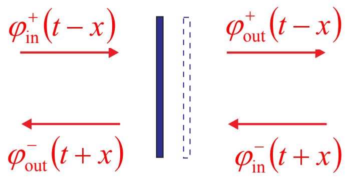
$$N_{cav} = \left(\frac{v}{c}\right)^2 \frac{1}{(\pi / F)^2 + (\Omega\tau - K\pi)^2}$$

Orders of magnitudes

- Mechanical oscillation frequency $\Omega/2\pi \sim 10$ GHz
- Supra-conducting cavity $F \sim 10^9$ at low temperature
- Mechanical parameters $v/c \sim 10^{-9}$
  - velocity $v \sim 30$ cm/s
  - amplitude $a \sim 10^{-11}$m
  - acceleration $\Omega v \sim 10^{10}$m/s^2
- Photons radiated outside the cavity $N \sim 10$ photons/second
- Photons inside the cavity $N_{cav} \sim 1$

The perturbative approach used above breaks down when the accumulated phase velocity *Fv/c* approaches unity

Non perturbative calculations of phaseshifts



- Free fields decomposed over the two directions of propagation
- Scattering on the mirror
  - depends on the motion
  - S-matrix describes reflection and transmission amplitudes

$$\begin{bmatrix} \varphi_{\text{out}}^{+}(t-q) \\ \varphi_{\text{out}}^{-}(t+q) \end{bmatrix} = S \begin{bmatrix} \varphi_{\text{out}}^{-}(t-q) \\ \varphi_{\text{out}}^{+}(t+q) \end{bmatrix}$$

- This scattering relation contains
  - ordinary phase shift experienced by the field for a mirror at rest
  - Doppler shift (change of frequency) for a mirror with a uniform velocity
  - full non perturbative phaseshift for an arbitrary motion *q(t)*

Harmonic motion

- Best representation of harmonic motion
  - homographic relation between the phase factors

$$e^{i\Omega v} = \frac{\alpha e^{i\Omega u} + \beta}{\beta^{*} e^{i\Omega u} + \alpha^{*}} \quad , \quad \alpha = e^{i\psi} \cosh\left(\frac{v}{c}\right) \quad , \quad \beta = e^{i\varphi} \sinh\left(\frac{v}{c}\right)$$

- Corresponds to a specific law of motion

$$\Omega q(t) = \psi - \arcsin\left(\tanh\left(\frac{v}{c}\right)\sin(\Omega t - \varphi)\right)$$

  - which reproduces ordinary sinusoidal motion for $v << c$
- Each motion is associated with a representative matrix

$$M = \begin{pmatrix} \alpha & \beta \\ \beta^{*} & \alpha^{*} \end{pmatrix}$$

Composition of motions

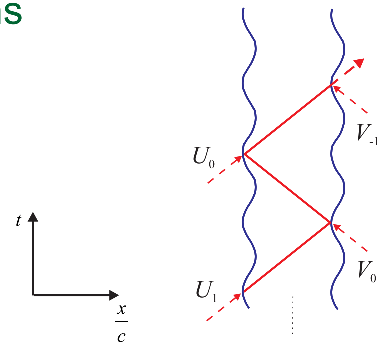
- Successive reflections are "composed" by multiplying their representative matrices

$u_0 \to v_0 \quad : \quad \text{matrix } M_1$

$v_0 \to u_1 \quad : \quad \text{matrix } M_2$

$u_0 \to u_1 \quad : \quad \text{matrix } M_2 M_1$

$u_0 \to u_n \quad : \quad \text{matrix } (M_2 M_1)^n$



Trajectory of light rays in space-time

- This leads to analytical expressions for
  - phaseshifts
  - radiated energy
  - spectrum

A. Lambrecht, M.-T. Jaekel & S. Reynaud, Eur. Phys. J. (1998)

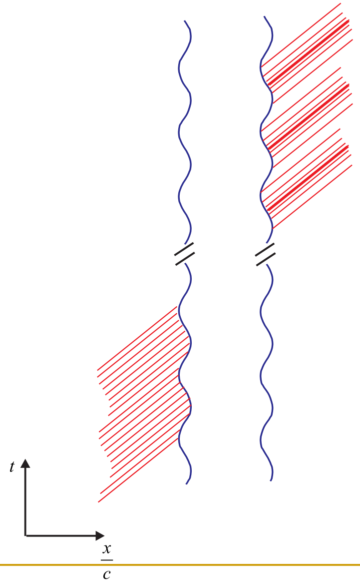
Attractors

Rays are attracted to stable periodic orbits and repelled from unstable periodic orbits

This leads to an energy build-up of the intracavity field which is particularly efficient for large values of

$$\eta = F\frac{v}{c}$$

This simulates the (unrealistic) situation of an harmonic motion which would approach the velocity of light

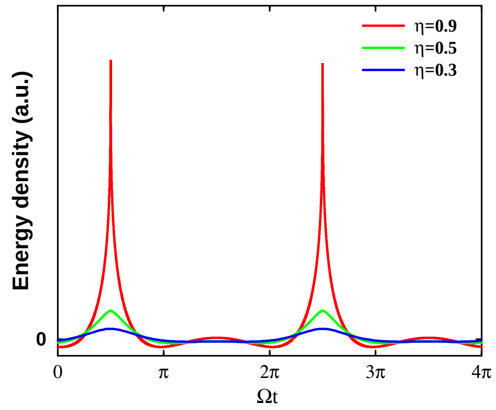


Energy build-up

Emitted energy density for different effective velocities

Oscillation threshold

$$\eta = F\frac{v}{c} = 1$$

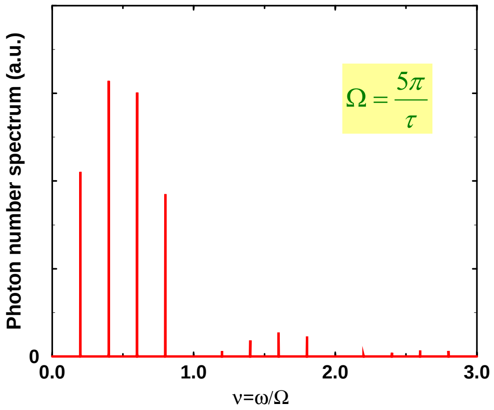


A. Lambrecht, M.-T. Jaekel & S. Reynaud, Eur. Phys. J. (1998)

Frequency up-conversion

Radiation spectrum for η=0.9

Specific signatures to be looked for in experiments



$$\Omega = \frac{5\pi}{\tau}$$

- Radiation at fractional multiples of Ω $\quad \omega = \frac{\pi}{\tau}, \frac{2\pi}{\tau}, \frac{3\pi}{\tau}, \frac{4\pi}{\tau}, \frac{6\pi}{\tau}, \ldots$
- No radiation at integer multiples of Ω

Towards an experimental observation ?

INSTITUTE OF PHYSICS PUBLISHING — JOURNAL OF OPTICS B: QUANTUM AND SEMICLASSICAL OPTICS

J. Opt. B: Quantum Semiclass. Opt. 7 (2005) S3–S10 — doi:10.1088/1464-4266/7/3/001

REVIEW ARTICLE

Electromagnetic pulses from an oscillating high-finesse cavity: possible signatures for dynamic Casimir effect experiments

Astrid Lambrecht

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- Emitted photons might be detected … (not easily) …
- Main problems :
  - excite the motion
  - isolate the few emitted photons from spurious radiation

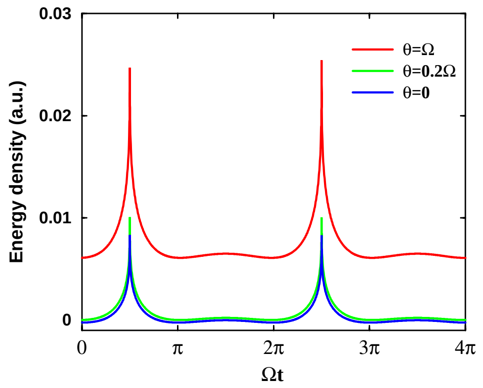
Temperature effects

$\theta = \frac{2\pi k_B T_{field}}{\hbar}$

Thermal fields also experience motion-induced effects

To observe vacuum effect we need $\theta \ll \Omega$

Ω~10GHz → T<10mK



Motion-induced radiation could be much more easily detected at high temperatures !

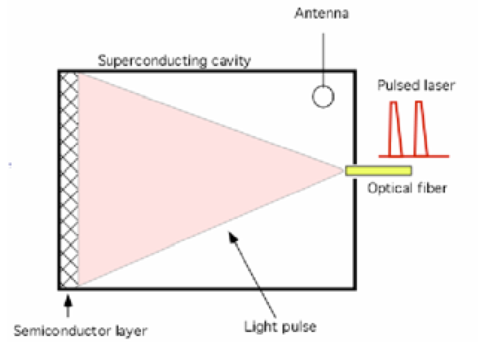
A. Lambrecht, M.-T. Jaekel & S. Reynaud, EuroPhys. Lett. 43 (1998)

A novel experimental approach…

New idea :

Suppress the mechanical motion of the mirror

Simulate the motion by a semiconductor layer excited by a laser at GigaHertz frequency



- New question :
  - does this simulation produce a good analogy to the case of moving mirrors ? (→ PhD François-Xavier Dezael)

Braggio, Bressi, Carugno *et al*, EuroPhys. Lett. 70 754 (2005)