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LEPTOP

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Abstract

This report consists of three parts. In Chapter 1 we present a brief description of the LEPTOP approach for the calculation of the radiative corrections in the Minimal Standard Model. The approach is based on the one-loop approximation with respect to the genuinely electroweak interaction for which simple explicit analytical formulae are valid. Starting from G_μ, m_Z and $\bar{\alpha} \equiv \alpha(m_Z^2)$ as the main input parameters, we consider all observables of the Z boson decays and the mass of the W boson on the same footing. The dressing with gluonic corrections is performed by using the results of calculations published by other authors.

In Chapter 2 theoretical uncertainties inherent to our approach and hence to the LEPTOP computer program are analyzed.

In Chapter 3 we describe the Fortran code of LEPTOP. This code can be obtained on request from rozanov@cppm.in2p3.fr or by anonymous ftp to cppm.in2p3.fr/pub/leptop.

This document can be accessed on `http://cppm.in2p3.fr/leptop/intro_leptop.html` by WWW users.

Chapter 1: LEPTOP BASIC EQUATIONS

1.1 Introduction

The program LEPTOP calculates radiative corrections to the observables of Z boson decay and of the W boson mass, m_W, in the Standard Model [1]. It is based on a set of explicit formulae for these observables, written in the papers [2] – [9].

Our aim is the study of the genuinely electroweak (e.-w.) interaction by comparing theoretical results with experimental data. In LEPTOP we separate the e.-w. interaction from the well known pure electromagnetic effects. Such separation is well defined in the one loop e.-w. approximation. Then we include all gluonic corrections (internal for hadron-free observables and both internal and external for the Z boson hadronic decays) known in the literature. The main result of our study (refs. [2]–[9]) is that within 1σ – 2σ accuracy experimental data are described by the e.-w. Born approximation which is universal for any modification of the Minimal Standard Model. The one-loop e.-w. corrections (that really specify the e.-w. theory) are barely visible at the level of LEP1, SLD, UA2 and CDF accuracy. We do not need to consider two-loop e.-w. corrections, except for enhanced gauge less top-quark contribution [10]-[13].

In LEPTOP we confine ourselves by considering effects larger than 10^{-5}.

Renormalization in LEPTOP. Loop calculations depend on parameter $\varepsilon = 4 - D$ in dimensional regularization (D is dimension of space-time), on the 't Hooft's scale parameter μ, on two bare gauge coupling constants, vacuum expectation value of the Higgs field and on the bare masses of quarks, leptons and the Higgs boson.[1] The dependence on ε and μ disappears if one rewrites the result of loop calculations in terms of renormalized coupling constants, physical masses, etc., or in terms of corresponding number of some other independent physical observables.

Input parameters. In LEPTOP we present the results of the loop calculations in terms of three most accurately known electroweak parameters [14], [15]:

$$m_Z = 91.1887(44) \quad \text{GeV} \tag{1.1}$$

$$G_\mu = 1.16639(2) \cdot 10^{-5} \quad \text{GeV}^{-2} \tag{1.2}$$

$$\bar{\alpha} \equiv \alpha(m_Z^2) = 1/128.87(12) \tag{1.3}$$

and three free unknown parameters: physical top quark mass m_t (on mass shell), Higgs mass m_H and strong coupling constant $\hat{\alpha}_s$ at scale m_Z: $\hat{\alpha}_s \equiv \alpha_s(q^2 = m_Z^2)_{\overline{MS}}$; the latter enters through gluonic corrections. There are tiny effects of nonzero b-quark mass m_b, τ-lepton mass m_τ and c-quark mass m_c. All other quarks and leptons can be considered as mass less.

Note that G_μ, $\bar{\alpha}$ and $\hat{\alpha}_s$ are not directly observable quantities:

1) G_μ is a four-fermion coupling constant of the muon decay corrected by purely electromagnetic loops.

[1] In general they also depend on Kobayashi–Maskawa angles but in the limit $m_q^2/m_Z^2 \ll 1$ for all quarks $q \neq t$ this dependence vanishes for Z boson decay and m_W.

2) Electromagnetic coupling constant $\bar{\alpha}$ at scale $q^2 = m_Z^2$ includes running from the value α to $\bar{\alpha}$ due to the three lepton and five "light" quark loops calculated by formulae:

$$\bar{\alpha} = \frac{\alpha}{1 - \delta\alpha} \tag{1.4}$$

$$\delta\alpha = \frac{m_Z^2}{4\pi^2\alpha} \int_{thr}^{\infty} \frac{ds}{m_Z^2 - s} \sigma_{e^+e^-}(s) \tag{1.5}$$

where α is fine structure constant and $\sigma_{e^+e^-}$ is the cross section of e^+e^--annihilation, through one virtual photon, into leptons and light hadrons:

$$\delta\alpha = \delta\alpha_l + \delta\alpha_h = 0.0314 + 0.0282(9)\ . \tag{1.6}$$

The leptonic contribution

$$\delta\alpha_l = \frac{\alpha}{3\pi}\Sigma_l(\ln\frac{m_Z^2}{m_l^2} - \frac{5}{3}) = 0.0314(0) \tag{1.7}$$

is known to high accuracy. The hadronic contribution, $\delta\alpha_h$, is calculated by using eq. (1.5) with the experimental cross-section $\sigma_{e^+e^-\to h}(s)$ in the dispersion integral below $s = (40\mathrm{GeV})^2$ and parton result above $s = (40\mathrm{GeV})^2$ (see ref. [15]).

3) The strong coupling constant $\hat{\alpha}_s$ at scale $q^2 = m_Z^2$ is defined as renormalized coupling in the $\overline{MS}$ scheme of subtraction.

By using $\bar{\alpha}$ instead of α we automatically take into account the fact that the running weak coupling constants $\alpha_W(q^2)$ and $\alpha_Z(q^2)$ do not run actually in the region $|q^2| \leq m_Z^2$, while $\alpha(q^2)$ does run. So $\bar{\alpha}$ is relevant to electroweak corrections (not α in spite of its extremely high accuracy) [2], [7].

The outline of Chapter 1 is as follows: In sect. 2 we introduce the general notations and phenomenological expressions for observables including the final state interaction due to photon and gluon exchange up to the terms of the order of 10^{-5}. In sect. 3 we consider the $\bar{\alpha}$-Born approximation. One e.-w. loop corrections are presented in sect. 4 for hadron-free observables (sect. 4.1) and for Z boson decay into hadrons (sect. 4.2). Chapter 2 is devoted to the accuracy of the LEPTOP. In sect. 5 we make general remarks concerning various sources of uncertainties. In sect. 6 we consider uncertainties in the vector boson self-energies. Sect. 7 is devoted to the uncertainties in the hadronic Z decays. The procedure of estimating the total theoretical accuracy of LEPTOP is described in sect. 8. Appendix A presents the flowchart of LEPTOP. Appendix B contains the full list of our papers in which the LEPTOP approach was developed.

1.2 Phenomenology and notations

It is useful to define the electroweak angle θ ($\sin\theta \equiv s$, $\cos\theta \equiv c$) in terms of three basic parameters G_μ , m_Z and $\bar{\alpha}$:

$$s^2c^2 = \frac{\pi\bar{\alpha}}{\sqrt{2}G_\mu m_Z^2} \tag{1.8}$$

(for earlier references see [16]). Solving eq. (1.8) with experimental values eqs. (1.1)–(1.3) we get

$$\begin{aligned} s^2 &= 0.23117(33) \\ c &= 0.87683(19) \end{aligned} \tag{1.9}$$

In LEPTOP the amplitude for Z boson decay into fermion-antifermion pair $f\bar{f}$ is written in one e.-w. loop approximation in the following form

$$\begin{aligned} M(Z \to f\bar{f}) &= \frac{1}{2}\bar{f} Z_\mu \bar{\psi}_f (\gamma_\mu g_{Vf} + \gamma_\mu\gamma_5 g_{Af})\psi_f \ , \\ \bar{f}^2 &= 4\sqrt{2} G_\mu m_Z^2 = 0.54866(8) \end{aligned} \tag{1.10}$$

In this amplitude we neglect such structures as the weak magnetic moment induced by e.-w. interaction. These structures are of the order of $(\alpha/\pi s^2)(m_f/m_Z)^2$, where m_f is the fermion mass, that is far beyond the accuracy of the present and future experiments; even for b-quark it is $\sim 10^{-5}$.

By definition the constants $g_{V,A}$ do not include the contribution from the interaction in the final state due to the exchange of gluons (for quarks) and photons (for quarks and leptons). The final state interaction includes also the processes of photon and gluon emission. Corresponding corrections have nothing to do with electroweak corrections and can be written separately.

Leptonic widths

For the decay into charged leptons $l\bar{l}$ we explicitly take into account the final state QED interaction factor

$$\Gamma_l = 4\Gamma_0[(g_V^l)^2(1 + \frac{3}{4\pi}\bar{\alpha}) + (g_A^l)^2(1 + \frac{3}{4\pi}\bar{\alpha} - 6\frac{m_l^2}{m_Z^2})] \tag{1.11}$$

where

$$\Gamma_0 = \frac{1}{24\sqrt{2}\pi} G_\mu m_Z^3 = 82.945(12)\text{MeV} \ . \tag{1.12}$$

Second order QED corrections $(\frac{\bar{\alpha}}{\pi})^2 \sim 10^{-6}$ are neglected. The term proportional to leptonic mass squared in eq. (1.11) is also negligible for $l = e, \mu$ (so we put $m_e = m_\mu = 0$) and is barely visible only for $l = \tau$ ($m_\tau^2/m_Z^2 = 3.8 \cdot 10^{-4}$).

For the neutrino decay

$$\begin{aligned} \Gamma_\nu &= 8 g_\nu^2 \Gamma_0 \ , \\ g_\nu &= g_{V\nu} = g_{A\nu} \ . \end{aligned} \tag{1.13}$$

Hadronic widths

For the decay into light quarks $q = u, d, s$ we neglect small effects due to nonzero quark mass (i.e. we put $m_u = m_d = m_s = 0$) and take into account the final state gluon exchange up to the third order [17], [18], [19], [20], the final state one photon exchange and the interference of the photon and gluon exchange [21]. These corrections are slightly different for vector and axial channels.

For the decay into quarks we have

$$\Gamma_q = \Gamma(Z \to q\bar{q}) = 12[g_{Aq}^2 R_{Aq} + g_{Vq}^2 R_{Vq}]\Gamma_0 \tag{1.14}$$

where factors $R_{A,V}$ are due to the final state interaction. (In our previous papers we used letter G instead of R for these factors). According to [18], [19], [20]

$$\begin{aligned} R_{Vq} &= 1 + \frac{\hat{\alpha}_s}{\pi} + \frac{3}{4}Q_q^2\frac{\bar{\alpha}}{\pi} - \frac{1}{4}Q_q^2\frac{\bar{\alpha}}{\pi}\frac{\hat{\alpha}_s}{\pi} + \\ &+ [1.409 + (0.065 + 0.015\ln t)\frac{1}{t}](\frac{\hat{\alpha}_s}{\pi})^2 - 12.77(\frac{\hat{\alpha}_s}{\pi})^3 + 12\frac{\hat{m}_q^2}{m_Z^2}\frac{\hat{\alpha}_s}{\pi}\delta_{vm} \end{aligned} \tag{1.15}$$

$$R_{Aq} = R_{Vq} - (2T_{3q})[I_2(t)(\frac{\hat{\alpha}_s}{\pi})^2 + I_3(t)(\frac{\hat{\alpha}_s}{\pi})^3] -$$

$$-12\frac{\hat{m}_q^2}{m_Z^2}\frac{\hat{\alpha}_s}{\pi}\delta_{vm} - 6\frac{\hat{m}_q^2}{m_Z^2}\delta_{am}^1 - 10\frac{\hat{m}_q^2}{m_t^2}(\frac{\hat{\alpha}_s}{\pi})^2\delta_{am}^2 \tag{1.16}$$

where $\hat{m}_q$ is running quark mass (see below),

$$\delta_{vm} = 1 + 8.7(\frac{\hat{\alpha}_s}{\pi}) + 45.15(\frac{\hat{\alpha}_s}{\pi})^2, \tag{1.17}$$

$$\delta_{am}^1 = 1 + 3.67(\frac{\hat{\alpha}_s}{\pi}) + (11.29 - \ln t)(\frac{\hat{\alpha}_s}{\pi})^2, \tag{1.18}$$

$$\delta_{am}^2 = \frac{8}{81} + \frac{\ln t}{54}, \tag{1.19}$$

$$I_2(t) = -3.083 - \ln t + \frac{0.086}{t} + \frac{0.013}{t^2} \ , \tag{1.20}$$

$$\begin{aligned} I_3(t) &= -15.988 - 3.722\ln t + 1.917\ln^2 t \ , \\ t &= m_t^2/m_Z^2 \ . \end{aligned} \tag{1.21}$$

The terms of the order of $(\frac{\hat{\alpha}_s}{\pi})^3$ due to diagrams with three gluons in the intermediate state have been calculated for R_{Vq} in ref.[20]. They have small numerical coefficients so that the net effect is of the order of 10^{-5} and we have omitted these terms in expansion (1.15).

For $Z \to b\bar{b}$ decay the nonzero b-quark mass is not negligible and produces contribution of the order of 1 MeV to Γ_b (i.e. of the order of 0.5–0.3%). Gluonic corrections effectively change the pole mass $m_b \simeq 4.7$GeV to the running mass at scale m_Z : $m_b \to \hat{m}_b(m_Z)$. We calculate $\hat{m}_b$ in terms of pole mass m_b , $\hat{\alpha}_s(m_Z)$ and $\hat{\alpha}_s(m_b)$ using standard two loops equation in $\overline{MS}$ scheme (see e.g. ref. [22]).

For $Z \to c\bar{c}$ decay the running mass $\hat{m}_c(m_Z)$ is expected to be of the order of 0.5 GeV and corresponding contribution to Γ_c is of the order of 0.05 MeV. Actually we take into account this tiny effect in LEPTOP because it is included in the other computer codes (ZFITTER).

In connection with Γ_c, let us note that the term $I_2(t)$ given by eq. (1.20) contains interference terms which appear at the order $(\frac{\hat{\alpha}_s}{\pi})^2$, hence its negative sign. These terms have three types of final states: one quark pair, one quark pair plus gluon, two quark pairs. The last final state constitutes about 5 % of the I_2 and at the present level of experimental accuracy is negligibly small. However, in principle, it calls for special care, especially when the two pairs consist of quarks of different flavor, e.g. $b\bar{b}c\bar{c}$. Such mixed final states should be a subject of special negotiations between theorists and experimentalists.

Asymmetries

a) Forward-backward asymmetry into $f\bar{f}$ channel is given by the equation

$$A_{FB}^{f\bar{f}} = \frac{3}{4} A_e A_f \tag{1.22}$$

where for light fermions

$$A_f = \frac{2 g_{Af} g_{Vf}}{(g_{Af})^2 + (g_{Vf})^2} \quad . \tag{1.23}$$

For b-quark we take into account the effect of nonzero mass:

$$A_b = \frac{2 g_{Ab} g_{Vb}}{[v^2 g_{Ab}^2 + \frac{1}{2}(3 - v^2) g_{Vb}^2]} v \ , \tag{1.24}$$

where v is the b-quark velocity:

$$v = \sqrt{1 - \frac{4 m_b^2(m_Z^2)}{m_Z^2}} \quad . \tag{1.25}$$

As was already mentioned, it is impossible to separate a given quark channel from another one unambiguously, starting from the order $(\frac{\hat{\alpha}_s}{\pi})^2$, just due to the possibility of creating an additional pair of "foreign" quarks. So we prefer not to consider terms of the order of $(\frac{\alpha_s}{\pi})^2$ in asymmetries at all. In this approximation the ratio g_V/g_A is not renormalized by the final state interaction.

b) Longitudinal polarization of τ-lepton

$$P_\tau = -A_\tau \tag{1.26}$$

c) The relative difference of total cross-section at Z peak for left- and right-handed electrons colliding with unpolarized positron beam

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = A_e \tag{1.27}$$

Other observables are defined in terms of Γ_q and Γ_l by equations:

a) Hadronic width (up to very small corrections) is given by the sum of five quark channels:

$$\Gamma_h = \Gamma_u + \Gamma_d + \Gamma_c + \Gamma_s + \Gamma_b \tag{1.28}$$

b) Total width:

$$\Gamma_Z = \Gamma_h + \Gamma_e + \Gamma_\mu + \Gamma_\tau + 3\Gamma_\nu \tag{1.29}$$

c) Peak cross section of e^+e^--annihilation into hadrons:

$$\sigma_h = \frac{12\pi}{M_Z^2}\frac{\Gamma_e\Gamma_h}{\Gamma_Z^2} \tag{1.30}$$

d) Ratios

$$R_c = \frac{\Gamma_c}{\Gamma_h}\ , \quad R_b = \frac{\Gamma_b}{\Gamma_h}\ , \quad R_l = \frac{\Gamma_h}{\Gamma_l} \quad . \tag{1.31}$$

1.3 The $\bar{\alpha}$–Born approximation

If we write the expressions for observables in the Born approximation in terms of s and c we automatically include purely electromagnetic interaction. Thus we get:

$$(m_W/m_Z)^B = c \tag{1.32}$$

$$(g_{Af})^B = T_{3f} \tag{1.33}$$

$$(g_{Vf}/g_{Af})^B = 1 - 4|Q_f|s^2 \tag{1.34}$$

Substituting $(g_{V,A})^B$ into equations for the widths and asymmetries from the previous section we calculate the whole set of observables in Z decay in the Born approximation for electroweak interaction, but take into account trivial pure electromagnetic and gluonic interaction in the final state and the effect of the running of e.-m. coupling constant. Rather luckily for LEPTOP these equations reproduce the precision experimental data for Z decay and for m_W (Jan. 95) with the accuracy $1\sigma - 1.5\sigma$ [6], [7], [23]. (Note that the final state interaction factors (eqs. (1.15)– (1.16)) include a lot of very small terms. One can neglect mass terms, as well as the difference between the vector and axial channels eq. (1.16). Such simplified Born approximation will still reproduce the data with practically the same accuracy).

It is important that the Born approximation for "low"–energy observables (parameters of Z boson decay and m_W) is the same for very different models, hence the present experimental confirmation of the $\bar{\alpha}$-Born approximation is not sufficient to prove the validity of the MSM or to choose between different generalizations.

1.4 Electroweak loop corrections

1.4.1 Hadron-free observables

For hadron-free observables we write the result of one-loop e.-w. calculations in the form suggested in ref. [2]:

$$m_W/m_Z = c + \frac{3c}{32\pi s^2(c^2 - s^2)}\bar{\alpha}V_m(t,h)$$

$$g_{Al} = -1/2 - \frac{3}{64\pi s^2 c^2}\bar{\alpha} V_A(t,h)$$
$$R = g_{Vl}/g_{Al} = 1 - 4s^2 + \frac{3}{4\pi(c^2-s^2)}\bar{\alpha} V_R(t,h) \tag{1.35}$$
$$g_\nu = 1/2 + \frac{3}{64\pi s^2 c^2}\bar{\alpha} V_\nu(t,h)$$

where $t = m_t^2/m_Z^2$, $h = m_H^2/m_Z^2$ and functions $V_i(t,h)$ are normalized by the condition

$$V_i(t,h) \simeq t \tag{1.36}$$

at $t \gg 1$.

Each function V_i is a sum of five terms

$$V_i(t,h) = t + T_i(t) + H_i(h) + C_i + \delta V_i(t,h) \tag{1.37}$$

The functions $t + T_i(t)$ are due to (t,b) doublet contribution to self-energies of the vector bosons, $H_i(h)$ is due to $W^\pm, Z$ and H loops, the constants C_i include light fermion contribution both to self energies, vertex and box diagrams.

To give explicit expressions for $T_i(t)$ and $H_i(h)$ it is convenient to introduce three auxiliary functions $F_t(t)$, $F_h(h)$ and $F_h'(h)$ (see subsection 4.3 for their expressions).

The equations for $T_i(t)$ and $H_i(h)$ have the form [2]:

$$\underline{i = m}$$

$$\begin{aligned}
T_m(t) &= (\frac{2}{3} - \frac{8}{9}s^2)\ln t - \frac{4}{3} + \frac{32}{9}s^2 + \\
&+ \frac{2}{3}(c^2 - s^2)(\frac{t^3}{c^6} - \frac{3t}{c^2} + 2)\ln \mid 1 - \frac{c^2}{t} \mid + \\
&+ \frac{2}{3}\frac{c^2-s^2}{c^4}t^2 + \frac{1}{3}\frac{c^2-s^2}{c^2}t + [\frac{2}{3} - \frac{16}{9}s^2 - \frac{2}{3}t - \frac{32}{9}s^2 t]F_t(t)\,; \\
H_m(h) &= -\frac{h}{h-1}\ln h + \frac{c^2 h}{h-c^2}\ln\frac{h}{c^2} - \frac{s^2}{18c^2}h - \frac{8}{3}s^2 + \\
&+ (\frac{h^2}{9} - \frac{4h}{9} + \frac{4}{3})F_h(h) - \\
&- (c^2 - s^2)(\frac{h^2}{9c^4} - \frac{4}{9}\frac{h}{c^2} + \frac{4}{3})F_h(\frac{h}{c^2}) + \\
&+ (1.1205 - 2.59\delta s^2)\,;
\end{aligned} \tag{1.38}$$

where $\delta s^2 = 0.23117 - s^2$ (note the sign!).

$$\underline{i = A}$$

$$T_A(t) = \frac{2}{3} - \frac{8}{9}s^2 + \frac{16}{27}s^4 - \frac{1 - 2tF_t(t)}{4t-1} +$$

$$+(\frac{32}{9}s^4 - \frac{8}{3}s^2 - \frac{1}{2})[\frac{4}{3}tF_t(t) - \frac{2}{3}(1+2t)\frac{1-2tF_t(t)}{4t-1}] \tag{1.39}$$

$$H_A(h) = \frac{c^2}{1-c^2/h}\ln\frac{h}{c^2} - \frac{8h}{9(h-1)}\ln h +$$

$$+(\frac{4}{3} - \frac{2}{3}h + \frac{2}{9}h^2)F_h(h) - (\frac{4}{3} - \frac{4}{9}h + \frac{1}{9}h^2)F'_h(h) -$$

$$-\frac{1}{18}h + [0.7751 + 1.07\delta s^2] \; .$$

$\underline{i = R}$

$$T_R(t) = \frac{2}{9}\ln t + \frac{4}{9} - \frac{2}{9}(1+11t)F_t(t) \; ;$$

$$H_R(h) = -\frac{4}{3} - \frac{h}{18} + \frac{c^2}{1-c^2/h}\ln\frac{h}{c^2} + \tag{1.40}$$

$$+(\frac{4}{3} - \frac{4}{9}h + \frac{1}{9}h^2)F_h(h) + \frac{h}{1-h}\ln h +$$

$$+(1.3590 + 0.51\delta s^2)$$

$\underline{i = \nu}$

$$T_\nu(t) = T_A(t)$$

$$H_\nu(h) = H_A(h) \tag{1.41}$$

The constants C_i are rather complicated functions of $\sin^2\theta$ and we present their numerical values at $s^2 = 0.23117 - \delta s^2$

$$C_m = -1.3500 + 4.13\delta s^2 \tag{1.42}$$

$$C_A = -2.2619 - 2.63\delta s^2 \tag{1.43}$$

$$C_R = -3.5041 - 5.72\delta s^2 \tag{1.44}$$

$$C_\nu = -1.1638 - 4.88\delta s^2 \tag{1.45}$$

Functions $\delta V_i(t,h)$ in eq. (1.37) are small corrections to V_i. They can be separated into five classes:

1. Corrections due to W-boson $\delta_W\alpha$ and t-quark $\delta_t\alpha$ polarization of e.-m. vacuum are traditionally not included into the running of $\alpha(q^2)$. We also prefer to consider them together with electroweak corrections. This is especially reasonable because W contribution $\delta_W\alpha$ is gauge dependent. Here and for all other e.-w. corrections we use 't Hooft–Feynman gauge. The corrections $\delta_W\alpha$ and $\delta_t\alpha$ were neglected in ref. [2] and were introduced in ref. [7].

$$\delta_1 V_m(t,h) = -\frac{16}{3}\pi s^4\frac{1}{\alpha}(\delta_W\alpha + \delta_t\alpha) = -0.055 \;\; , \tag{1.46}$$

$$\delta_1 V_R(t,h) = -\frac{16}{3}\pi s^2 c^2 \frac{1}{\alpha}(\delta_W \alpha + \delta_t \alpha) = -0.181 \ , \tag{1.47}$$

$$\delta_1 V_A(t,h) = \delta_1 V_\nu(t,h) = 0 \ , \tag{1.48}$$

where

$$\frac{\delta_W \alpha}{\alpha} = \frac{1}{2\pi}[(3+4c^2)(1-\sqrt{4c^2-1}\arcsin\frac{1}{2c}) - \frac{1}{3}] = 0.0686 \ , \tag{1.49}$$

$$\frac{\delta_t \alpha}{\alpha} = -\frac{4}{9\pi}[(1+2t)F_t(t) - \frac{1}{3}] \simeq -\frac{4}{45\pi}\frac{1}{t} + \ldots \simeq -0.00768 \ . \tag{1.50}$$

(Here and in the equations (1.51) – (1.69) we use $m_t = 175$ GeV for numerical estimates.)

2. Corrections of the order of $\bar{\alpha}\hat{\alpha}_s$ due to the gluon exchange in the quark e.-w. loops [24] (see also [3]). For the two generations of light quarks ($q = u, d, s, c$) it gives:

$$\delta_2^q V_m(t,h) = 2 \cdot [\frac{4}{3}(\frac{\hat{\alpha}_s(m_Z)}{\pi})(c^2 - s^2)\ln c^2] = (\frac{\hat{\alpha}_s(m_Z)}{\pi})(-0.377) \tag{1.51}$$

$$\delta_2^q V_A(t,h) = \delta_2^q V_\nu(t,h) = 2 \cdot [\frac{4}{3}(\frac{\hat{\alpha}_s(m_Z)}{\pi})(c^2 - s^2 + \frac{20}{9}s^4)] = (\frac{\hat{\alpha}_s(m_Z)}{\pi})(1.750) \tag{1.52}$$

$$\delta_2^q V_R(t,h) = 0 \tag{1.53}$$

The result of calculation for third generation is rather complicated function of top-mass:

$$\delta_2^t V_m(t,h) = \frac{4}{3}(\frac{\hat{\alpha}_s(m_t)}{\pi})\{tA_1(\frac{1}{4t}) + (1 - \frac{16}{3}s^2)tV_1(\frac{1}{4t}) + (\frac{1}{2} - \frac{2}{3}s^2)\ln t$$
$$- 4(1 - \frac{s^2}{c^2}) \times tF_1(\frac{c^2}{t}) - 4\frac{s^2}{c^2}tF_1(0)\} \tag{1.54}$$

$$\delta_2^t V_A(t,h) = \delta_2^t V_\nu(t,h) = \frac{4}{3}(\frac{\hat{\alpha}_s(m_t)}{\pi})\{tA_1(\frac{1}{4t}) - \frac{1}{4}A_1^{'}(\frac{1}{4t}) +$$
$$+ (1 - \frac{8}{3}s^2)^2[tV_1(\frac{1}{4t}) - \frac{1}{4}V_1'(\frac{1}{4t})] + (\frac{1}{2} - \frac{2}{3}s^2 + \frac{4}{9}s^4) - 4tF_1(0)\} \tag{1.55}$$

$$\delta_2^t V_R(t,h) = \frac{4}{3}(\frac{\hat{\alpha}_s(m_t)}{\pi})\{tA_1(\frac{1}{4t}) - \frac{5}{3}tV_1(\frac{1}{4t}) - 4tF_1(0) + \frac{1}{6}\ln t\}, \tag{1.56}$$

where

$$\hat{\alpha}_s(m_t) = \frac{\hat{\alpha}_s(M_Z)}{1 + \frac{23}{12\pi}\hat{\alpha}_s(M_Z)\ln t} \tag{1.57}$$

Note that $\delta_2 V_i$, unlike V_i themselves, do not depend on m_H. The functions $V_1(r)$, $A_1(r)$ and $F_1(x)$ have a very complicated form and were calculated in ref. [24]. For our purpose we can use their rather simple Taylor expansion for small values of their arguments (we have added cubic terms to the expansion presented in ref. [24]):

$$V_1(r) = r[4\zeta(3) - \frac{5}{6}] + r^2\frac{328}{81} + r^3\frac{1796}{25\times 27} + ... \quad (1.58)$$

$$A_1(r) = [-6\zeta(3) - 3\zeta(2) + \frac{21}{4}] + r[4\zeta(3) - \frac{49}{18}] + r^2\frac{689}{405} + r^3\frac{3382}{7\times 25\times 27} + ... \quad (1.59)$$

$$F_1(x) = [-\frac{3}{2}\zeta(3) - \frac{1}{2}\zeta(2) + \frac{23}{16}] + x[\zeta(3) - \frac{1}{9}\zeta(2) - \frac{25}{72}] + \\ + x^2[\frac{1}{8}\zeta(2) + \frac{25}{3\times 64}] + x^3[\frac{1}{30}\zeta(2) + \frac{5}{72}] + ... \quad (1.60)$$

where $\zeta(2) = \pi^2/6, \zeta(3) = 1.2020569....$

By summing contributions of (1.54)-(1.56) and using expansions (1.58)-(1.60) one obtains up to terms $0(1/t^3)$:

$$\delta_2^t V_m(t,h) = (\frac{\hat{\alpha}_s(m_t)}{\pi})[-2.86t + 0.46\ln t - 1.540 - \frac{0.68}{t} - \frac{0.21}{t^2}] = \frac{\hat{\alpha}_s(m_t)}{\pi}(-11.67) \quad (1.61)$$

$$\delta_2^t V_A(t,h) = \delta_2^t V_\nu(t,h) = (\frac{\hat{\alpha}_s(m_t)}{\pi})[-2.86t + 0.493 - \frac{0.19}{t} - \frac{0.05}{t^2}] = \frac{\hat{\alpha}_s(m_t)}{\pi}(-10.10) \quad (1.62)$$

$$\delta_2^t V_R(t,h) = (\frac{\hat{\alpha}_s(m_t)}{\pi})[-2.86t + 0.22\ln t - 1.513 - \frac{0.42}{t} - \frac{0.08}{t^2}] = \frac{\hat{\alpha}_s(m_t)}{\pi}(-11.88) \quad (1.63)$$

As these formulas are valid for $m_t > m_Z$, in order to go to the region $m_t < m_Z$ we either put $\delta_2^t V_i = 0$ or use mass less limit in which $\delta_2^t V_i = \frac{1}{2}\delta_2^q V_i$. In any case this region gives tiny contribution into global fit.

3. Corrections of the order of $\bar{\alpha}\hat{\alpha}_s^2$ were calculated for leading term $\bar{\alpha}\hat{\alpha}_s^2 t$ only [25]

$$\delta_3 V_i(t,h) \simeq -(2.38 - 0.18N_f)\hat{\alpha}_s^2(m_t)t \simeq -1.48\hat{\alpha}_s^2(m_t)t = -0.07 \quad (1.64)$$

for $N_f = 5$ light flavors. (For numerical estimate we use $\hat{\alpha}_s(m_Z) = 0.125$.)

4. The leading correction of the order of $\bar{\alpha}^2 t^2$ which originates from the second order Yukawa interaction was calculated in ref. [10]-[13]

$$\delta_4 V_i(t,h) = -\frac{\bar{\alpha}}{16\pi s^2 c^2} A(\frac{h}{t}) \cdot t^2 \quad (1.65)$$

Function $A(\frac{h}{t})$ is given in the table 1.1 [12]-[13] for $m_H/m_t < 4$. We use expansion from [10]-[11] for $m_H/m_t > 4$:

$$A(\frac{h}{t}) = -[\frac{49}{4} + \pi^2 + \frac{27}{2}\ln r + \frac{3}{2}\ln^2 r + \frac{1}{3}r(2 - 12\pi^2 + 12\ln r - 27\ln^2 r) +$$

$$+\frac{1}{48}r^2(1613-240\pi^2-1500\ln r-720\ln^2 r)], \qquad (1.66)$$

where $r=t/h$. For $m_t=175$ GeV and $m_H=300$ GeV one has $A=8.9$ and $\delta_4 V_i(t,h)=-0.11$.

Additional terms of the order $\bar{\alpha}^2 t^2$ are needed when dealing with 2 electroweak loops corrections. They are parametrically the same as Barbieri correction. These terms are not universal.

$$\delta_4' V_m(t,h)=\frac{\bar{\alpha}}{16\pi s^2 c^2}\frac{3c^2(3-10s^2)}{4(c^2-s^2)^2}\cdot t^2$$

$$\delta_4' V_A(t,h)=\frac{\bar{\alpha}}{16\pi s^2 c^2}\frac{9}{4}\cdot t^2$$

$$\delta_4' V_R(t,h)=-\frac{\bar{\alpha}}{16\pi s^2 c^2}\frac{3c^2 s^2}{(c^2-s^2)^2}\cdot t^2$$

5. In the second order in e.-w. interactions there appears quadratic dependence on the Higgs mass [26].

$$\delta_5 V_m=\frac{\bar{\alpha}}{24\pi}(\frac{m_H^2}{m_Z^2})\times\frac{0.747}{c^2}=0.0011 \qquad (1.67)$$

$$\delta_5 V_A=\delta_5 V_\nu=\frac{\bar{\alpha}}{24\pi}(\frac{m_H^2}{m_Z^2})\times\frac{1.199}{s^2}=0.0057 \qquad (1.68)$$

$$\delta_5 V_R=-\frac{\bar{\alpha}}{24\pi}(\frac{m_H^2}{m_Z^2})\frac{c^2-s^2}{s^2c^2}\times 0.973=-0.0032\ . \qquad (1.69)$$

(For numerical estimates we use $m_H=300$ GeV.)

1.4.2 Z boson decay into hadrons

For the partial width of the Z decay into a pair of quarks $q\bar{q}$ ($q=u,d,s,c,b$) we use the equation (1.14)

$$\Gamma_q=12[g_{Aq}^2 R_{Aq}+g_{Vq}^2 R_{Vq}]\Gamma_0+\delta\Gamma_q\ ,$$

where e.-w. radiative corrections are included into g_{Vq} and g_{Aq}

$$g_{Aq}=T_{3q}[1+\frac{3\bar{\alpha}}{32\pi s^2c^2}V_{Aq}(t,h)] \qquad (1.70)$$

$$g_{Vq}/g_{Aq}=1-4|Q_q|s^2+\frac{3|Q_q|}{4\pi(c^2-s^2)}\bar{\alpha}V_{Rq}(t,h)\ . \qquad (1.71)$$

The functions $V_{Aq}(t,h)$ and $V_{Rq}(t,h)$ in the one-loop e.-w- approximation are related to the functions $V_A(t,h)$ and $V_R(t,h)$ from leptonic decays [5]:

$$V_{Au}(t,h)=V_{Ac}(t,h)=V_A(t,h)+\frac{128\pi s^3c^3}{3\bar{\alpha}}(F_{Al}+F_{Au}) \qquad (1.72)$$

$$V_{Ad}(t,h) = V_{As}(t,h) = V_A(t,h) + \frac{128\pi s^3 c^3}{3\bar{\alpha}}(F_{Al} - F_{Ad}) \tag{1.73}$$

$$V_{Ru}(t,h) = V_{Rc}(t,h) = V_R(t,h) + \frac{16\pi sc(c^2 - s^2)}{3\bar{\alpha}} \times \\ \times[F_{Vl} - (1-4s^2)F_{Al} + \frac{3}{2}(-(1-\frac{8}{3}s^2)F_{Au} + F_{Vu})] \tag{1.74}$$

$$V_{Rd}(t,h) = V_{Rs}(t,h) = V_R(t,h) + \frac{16\pi sc(c^2 - s^2)}{3\bar{\alpha}} \times \\ \times[F_{Vl} - (1-4s^2)F_{Al} + 3((1-\frac{4}{3}s^2)F_{Ad} - F_{Vd})] \tag{1.75}$$

where:

$$F_{Al} = \frac{\bar{\alpha}}{4\pi}(3.0088 + 16.4\delta s^2)\ , \tag{1.76}$$

$$F_{Vl} = \frac{\bar{\alpha}}{4\pi}(3.1868 + 14.9\delta s^2)\ , \tag{1.77}$$

$$F_{Au} = -\frac{\bar{\alpha}}{4\pi}(2.6792 + 14.7\delta s^2)\ , \tag{1.78}$$

$$F_{Vu} = -\frac{\bar{\alpha}}{4\pi}(2.7319 + 14.2\delta s^2)\ , \tag{1.79}$$

$$F_{Ad} = \frac{\bar{\alpha}}{4\pi}(2.2212 + 13.5\delta s^2)\ , \tag{1.80}$$

$$F_{Vd} = \frac{\bar{\alpha}}{4\pi}(2.2278 + 13.5\delta s^2)\ . \tag{1.81}$$

The five digit accuracy of the above numbers is purely arithmetical. The physical uncertainties (caused by the neglect of the higher loop corrections) are at the level of the third digit.

The difference $V_{iq} - V_i$ (where $i = A, R$) is due to different e.-w. corrections to the vertices $Zq\bar{q}$ and $Zl\bar{l}$.

The oblique corrections of the order of $\hat{\alpha}_s$, $\hat{\alpha}_s^2 t$ and $\bar{\alpha}t^2$ to the $V_{Aq}(V_{Rq})$ are the same as in the case of V_A and V_R. But for Z-boson decay into pair $q\bar{q}$ there are also additional $\hat{\alpha}_s$ corrections to the vertices that have not been calculated yet. That brings additional uncertainty into the theoretical accuracy.

For $Z \to b\bar{b}$ decays we have to take into account corrections to the $Z \to b\bar{b}$ vertex which depend on t [27], [28]

$$V_{Ab}(t,h) = V_{Ad}(t,h) - \frac{8s^2c^2}{3(3-2s^2)}(\phi(t) + \delta\phi(t)), \tag{1.82}$$

$$V_{Rb}(t,h) = V_{Rd}(t,h) - \frac{4s^2(c^2-s^2)}{3(3-2s^2)}(\phi(t) + \delta\phi(t)) \tag{1.83}$$

where for $\phi(t)$ we use the following expansion [27]

$$\phi(t) = \frac{3-2s^2}{2s^2c^2}\Big\{t + c^2[2.88ln\frac{t}{c^2} - 6.716+$$

$$+\frac{1}{t}(8.368c^2\ln\frac{t}{c^2} - 3.408c^2) + \frac{1}{t^2}(9.126c^4\ln\frac{t}{c^2} + 2.26c^4) + \tag{1.84}$$

$$+\frac{1}{t^3}(4.043c^6\ln\frac{t}{c^2} + 7.41c^6) + ...\Big\}$$

and for $\delta\phi(t)$ we use the leading approximation calculated in ref. [28] and [10]-[13],

$$\delta\phi(t) = \frac{3-2s^2}{2s^2c^2}\left\{-\frac{\pi^2}{3}(\frac{\hat{\alpha}_s(m_t)}{\pi})t + \frac{1}{16s^2c^2}(\frac{\bar{\alpha}}{\pi})t^2\tau_b^{(2)}(\frac{h}{t})\right\} \quad , \tag{1.85}$$

where the function $\tau_b^{(2)}$ is given by the table 1.1 [12]-[13] for $m_H/m_t < 4$. For $m_H/m_t > 4$ we use the expansion [10]-[11]:

$$\tau_b^{(2)}(\frac{h}{t}) = \frac{1}{144}[311 + 24\pi^2 + 282\ln r + 90\ln^2 r - 4r(40 + 6\pi^2 + 15\ln r + 18\ln^2 r)+$$

$$+\frac{3}{100}r^2(24209 - 6000\pi^2 - 45420\ln r - 18000\ln^2 r)], \tag{1.86}$$

where $r = t/h$. For $m_t = 175$ GeV and $m_H = 300$ GeV

$$\tau_b^{(2)} = 1.245 \quad .$$

Recently the additional $\bar{\alpha}\hat{\alpha}_s$ corrections to Z-boson width were calculated by Czarnecki-Kuhn [29] and Harlander et al. [30]. They are:

$$\delta\Gamma_{u,c} = -0.000124\text{GeV}$$

for u and c quarks.

$$\delta\Gamma_{d,s} = -0.000173\text{GeV}$$

for d and s quarks.

$$\delta\Gamma_b = -0.00004\text{GeV}$$

for b quarks.

Asymmetries are calculated by using eqs. (1.22) – (1.27) with the loop corrected values of g_A and g_V.

1.4.3 Auxiliary functions F_t and F_h

The functions $F_t(t)$ and $F_h(h)$ are used in eqs. (1.38) – (1.40).

$$F_t(t) = \begin{cases} 2(1 - \sqrt{4t-1}\arcsin\frac{1}{\sqrt{4t}}) & , \quad 4t > 1 \\ \\ 2(1 - \sqrt{1-4t}\ln\frac{1+\sqrt{1-4t}}{\sqrt{4t}}) & , \quad 4t < 1 \end{cases} \tag{1.87}$$

$$F_h(h) = \begin{cases} 1 + (\frac{h}{h-1} - \frac{h}{2})\ln\ h + h\sqrt{1-\frac{4}{h}}\ln(\sqrt{\frac{h}{4}-1} + \sqrt{\frac{h}{4}}) & , \quad h > 4 \\ \\ 1 + (\frac{h}{h-1} - \frac{h}{2})\ln\ h - h\sqrt{\frac{4}{h}-1}\arctan\sqrt{\frac{4}{h}-1} & , \quad h < 4 \end{cases} \tag{1.88}$$

$$F_h'(h) = \begin{cases} -1 + \frac{h-1}{2}\ln\ h + (3-h)\sqrt{\frac{h}{h-4}}\ln(\sqrt{\frac{h}{4}-1} + \sqrt{\frac{h}{4}}) & , \quad h > 4 \\ \\ -1 + \frac{h-1}{2}\ln\ h + (3-h)\sqrt{\frac{h}{4-h}}\arctan(\sqrt{\frac{4-h}{h}}) & , \quad h < 4 \end{cases} \tag{1.89}$$

Table 1.1: Functions $A(\frac{m_H}{m_t})$ and $\tau^{(2)}(\frac{m_H}{m_t})$ from [12]-[13].

$\frac{m_H}{m_t}$	$A(\frac{m_H}{m_t})$	$\tau^{(2)}(\frac{m_H}{m_t})$
.00	.739	5.710
.10	1.821	4.671
.20	2.704	3.901
.30	3.462	3.304
.40	4.127	2.834
.50	4.720	2.461
.60	5.254	2.163
.70	5.737	1.924
.80	6.179	1.735
.90	6.583	1.586
1.00	6.956	1.470
1.10	7.299	1.382
1.20	7.617	1.317
1.30	7.912	1.272
1.40	8.186	1.245
1.50	8.441	1.232
1.60	8.679	1.232
1.70	8.902	1.243
1.80	9.109	1.264
1.90	9.303	1.293
2.00	9.485	1.330
2.10	9.655	1.373
2.20	9.815	1.421
2.30	9.964	1.475
2.40	10.104	1.533
2.50	10.235	1.595
2.60	10.358	1.661
2.70	10.473	1.730
2.80	10.581	1.801
2.90	10.683	1.875
3.00	10.777	1.951
3.10	10.866	2.029
3.20	10.949	2.109
3.30	11.026	2.190
3.40	11.098	2.272
3.50	11.165	2.356
3.60	11.228	2.441
3.70	11.286	2.526
3.80	11.340	2.613
3.90	11.390	2.700
4.00	11.436	2.788

Chapter 2: LEPTOP THEORETICAL UNCERTAINTIES

2.1 General remarks

With the increasing experimental accuracy of the precision measurements at LEP and SLC the question of the accuracy of the theoretical predictions becomes one of great importance. There are several major sources of uncertainties:

1. The degree of accuracy with which any given observable is extracted from the experimental data. This involves theoretical models and computer simulations (Monte Carlo etc.) which deal with purely electromagnetic and strong interactions, for instance, in extracting m_W from the $p\bar{p}$-collider data, or Z-observables (in particular for separate quark channels) from e^+e^--data. We call them "extraction" uncertainties.

2. Uncertainties in the input parameters ($\bar{\alpha}$, m_Z, G_μ, m_b, m_τ etc.), so called "parametric" uncertainties.

3. The degree of accuracy with which the theoretical expression for a given electroweak observable is derived within the MSM or some of its generalizations ("theoretical" uncertainties).

In this paper we shall mainly deal with the third kind of uncertainties (in the framework of the MSM), although for some of observables, such as R_b or A^b_{FB} or Q_{FB}, they may not be fully separable from those of the first kind. One also has to keep in mind that some of the physical quantities we shall discuss such as Γ_h, Γ_e, g_A, are, strictly speaking, not primary, but derived observables.

There are several types of observables with specific uncertainties for each type:

1. "hadron-free" observables: m_W (m_W/m_Z), Γ_l, g_A, g_V/g_A.

2. Observables of hadronic decays of Z boson.

3. Observables of the low-energy weak processes: νe-scattering, νN-scattering, atomic parity violation parameters.

Let us finish this introductory section with the estimates of the "parametric" uncertainties for the Z-decay parameters and the W boson mass. Among the three basic input parameters $\bar{\alpha}$,G_μ and m_Z the largest uncertainty comes from $\bar{\alpha} \equiv \alpha(m_Z) : \bar{\alpha}^{-1} = 128.87(12)$. The relative uncertainty in m_Z^2 is an order of magnitude smaller ($m_Z = 91.1887(44)$); that in G_μ is 50 times smaller ($G_\mu = 1.16639(2) \cdot 10^{-5}\mathrm{GeV}^{-2}$). Induced by the uncertainty in $\bar{\alpha}$ the absolute uncertainty in m_W/m_Z is close to $2 \cdot 10^{-4}$, that in g_V/g_A is close to $1 \cdot 10^{-3}$, in Γ_h it equals 0.8 MeV and in σ_h it is 0.001 nb. Finally, in Γ_b it is 0.15 MeV. All these uncertainties come mainly from the uncertainty of the $\bar{\alpha}$-Born approximation due to that in the parameter $s^2 \equiv \sin^2\theta = 0.2312(3)$.

Let us stress that relative parametric uncertainty in g_V/g_A is quite large: about 1.5%. The experimental accuracy with which g_V/g_A is measured is becoming close to the uncertainty due to $\delta\bar{\alpha}$. Improvement of the accuracy of $\bar{\alpha}$ calls for new measurement of $e^+e^- \to$ *hadrons* cross-section below J/ψ resonance. Measurement of $(g-2)_\mu$ with high accuracy will also help.

2.2 Uncertainties in V_i

Theoretical uncertainties come from as yet not calculated Feynman diagrams or not calculated terms in a given diagram. The smallest calculated terms in the W and Z self-energies are the virtual Higgs corrections of the order $\alpha_W^2 t^2$ [10]-[13], and the two gluon correction $\alpha_W \hat{\alpha}_s^2 t$ [25] to the top quark loop. These corrections produce universal shifts δV_i in the functions V_i which describe radiative corrections in the approach developed in [2]:

$$\delta V_i^{t^2} = -\frac{\bar{\alpha}}{\pi}\frac{A(m_h/m_t)t^2}{16s^2c^2} \tag{2.1}$$

$$\delta V_i^{\alpha_s^2} = -1.2\hat{\alpha}_s^2(m_t)t \tag{2.2}$$

Here and in what follows we denote by δ the corrections, while by Δ – the uncertainties. According to [31], still not completely calculated correction to (2.1), of the order of $\bar{\alpha}t$, can be numerically close to (2.1). The same, according to the literature, is valid for uncertainties in eq. (2.2). In order to have correct asymptotic behavior of the uncertainties at $t \gg 1$ we assume that uncertainties corresponding to expressions (2.1) and (2.2) are derived by multiplying both of them by $2/t$. For $m_H = 300$ GeV , $m_t = 175$ GeV and $\hat{\alpha}_s(m_Z) = 0.125$ our estimates are $\Delta V_i^{t^2} = \pm 0.06$, $\Delta V_i^{\alpha_s^2} = \pm 0.03$; corresponding uncertainties in the observables are presented in the Table 1.

2.3 Uncertainties in hadronic Z decays

In hadronic decays we separate decays into light quarks $q\bar{q}$ (where $q = u, d, s, c$) and the decay into $b\bar{b}$. Concerning Γ_q the next not yet calculated Feynman diagrams are the gluon corrections to the Zqq electroweak vertex "triangle" involving W or Z. The electroweak corrections to the vertices in the $Z \to qq$ decays are given by the following equation:

$$\delta\Gamma_q = 24sc\Gamma_0 2T_3^q\{(1-4|Q_f|s^2)F_{Vq} + F_{Aq}\} \ . \tag{2.3}$$

Substituting numbers in eq. (2.3) we get:

$$\Delta\Gamma_u = \Delta\Gamma_c = -1.9 \text{ MeV} \ , \tag{2.4}$$

$$\Delta\Gamma_d = \Delta\Gamma_s = -2.0 \text{ MeV} \ . \tag{2.5}$$

Taking into account 4 light quark flavors and multiplying the results (2.4) and (2.5) by $\hat{\alpha}_s(m_Z)/\pi$ we get the following estimate of the uncertainty in Z width into light quarks due to the gluonic perturbation of Zqq vertex:

$$\sum_1^4 \Delta\Gamma_q^{\bar{\alpha}\hat{\alpha}_s} = \pm 0.3 \text{ MeV} \tag{2.6}$$

For the $Z \to b\bar{b}$ decay the leading term $\sim \alpha_W \hat{\alpha}_s t$ [28], and the potentially next to the leading term $\sim \alpha_W \hat{\alpha}_s \ln t$ [32] come from virtual gluons in the ttW vertex triangle. Calculated in [28] the leading α_s correction to electroweak vertex lead to the following correction to Γ_b:

$$\Delta\Gamma_b^{\bar{\alpha}\hat{\alpha}_s t} = -\frac{\bar{\alpha}}{\pi}\Gamma_0\frac{(3-2s^2)}{2s^2c^2}(-\frac{\pi^2}{3})\frac{\hat{\alpha}_s(m_t)}{\pi}t = 0.7 \text{ MeV} \ , \tag{2.7}$$

where for numerical estimate we substitute $m_t = 175$ GeV.
Calculated in [32] $\hat{\alpha}_s$ correction to $\ln t$ term appears to be much smaller:

$$\delta\Gamma_b^{\bar{\alpha}\hat{\alpha}_s \ln t} = -\frac{\bar{\alpha}}{\pi}\Gamma_0\frac{(3-2s^2)^2}{16s^2c^2}\frac{7}{81}\frac{\hat{\alpha}_s(m_t)}{\pi}\ln t \simeq 0.002 \text{ MeV} \tag{2.8}$$

If we suppose that un-calculated terms are of the order of the last calculated one we can safely neglect this type of uncertainties.
Uncertainty in m_b also shows itself in Γ_b value. However, it is comparatively small. Even if we assume that m_b is known with accuracy ± 300 MeV, the shift in Γ_b is ± 0.17 MeV. There is a specific to $Z \to b\bar{b}$ decay virtual Higgs gauge-less correction to the vertex $\delta\phi(t,h)$ consisting of terms of the order $\alpha_W^2 t^2$, $\alpha_W^2 t \ln t$, $\alpha_W^2 t$ and so on. Only the first of them have been calculated [10]-[13] :

$$\delta\phi^{t^2} = \frac{3-2s^2}{2s^2c^2}\frac{\bar{\alpha}t^2}{16\pi s^2c^2}\tau_b^{(2)} \ . \tag{2.9}$$

If we consider this leading term as an upper bound of the "Higgs theoretical uncertainty" in $\Delta\phi$ than we get:

$$\Delta\Gamma_b^{t^2} = -\frac{\bar{\alpha}}{\pi}\Gamma_0\Delta\phi^{t^2} = -\frac{\bar{\alpha}}{\pi}\Gamma_0\frac{3-2s^2}{2s^2c^2}\frac{\bar{\alpha}t^2}{16\pi s^2c^2}\tau_b^{(2)} = 0.02 \text{ MeV} \ , \tag{2.10}$$

where we substitute $m_t = 175$ GeV , $m_H = 300$ GeV. This uncertainty in Γ_b is much smaller than those considered above and could be neglected. However, correction of the order of $\hat{\alpha}_s^2(m_t)t$ to $\phi(t)$, $\delta\phi^{\alpha_s^2}$, may lead to larger correction to Γ_b.
To estimate its value let us suppose that it's ratio to the calculated correction $\sim \hat{\alpha}_s t$ is equal to the ratio of the calculated corrections $\sim \hat{\alpha}_s^2 t$ and $\sim \hat{\alpha}_s t$ to functions V_i:

$$\frac{\delta\phi^{\alpha_s^2}}{\delta\phi^{\alpha_s}} = \frac{\delta_3 V_i}{\frac{\hat{\alpha}_s(m_t)}{\pi}(-2.86)t}, \tag{2.11}$$

$$\delta\phi^{\alpha_s^2} = -\frac{3-2s^2}{2s^2c^2}\frac{1.37\pi}{3}\hat{\alpha}_s^2(m_t)t. \tag{2.12}$$

Taking this estimate as a source of uncertainty in Γ_b, for $m_t = 175$ GeV we get: $\Delta\Gamma_b^{\alpha_s^2} = 0.1$ MeV, which dominates over (2.8) and (2.10).

2.4 Procedure to estimate total theoretical accuracy of LEPTOP

In order to estimate the total theoretical uncertainties for the observables we implement in LEPTOP the procedure proposed by D.Bardin and G.Passarino [33]. We choose several options to the preferred formulas used in LEPTOP and make variations of these formulas. Usually each option consists in the addition of an extra term corresponding to a rough guess of the value of the uncalculated higher order terms. We then make all possible combinations of these options, i.e. 2^n in total, where n is the number of options. Among all these 2^n combinations we locate those yielding the minimum and the maximum values of the observables and took as the estimate of the theoretical errors their deviations from the central values.

We consider the following options:

`Options 1,2`	Correction $\delta V_i^{t^2}$ given by eq. (2.1) multiplied by $2/t$ is added to (option 1) (or subtracted from (option 2)) the three functions V_A , V_m and V_R.
`Options 3,4`	The same as in options 1,2 with $\delta V_i^{\alpha_s^2}$ from eq. (2.2).
`Options 5,6`	The expression for Γ_q is modified by adding (option 5) or subtracting (option 6) $\sum_1^4 \Delta\Gamma_q = 0.3$ MeV in order to take into account gluon corrections to electroweak Zqq triangle vertices.
`Option 7`	Correction $\delta\phi^{\alpha_s^2}$ eq. (2.12) is added to function $\phi(t)$.

Table 2.1: *The effect of the working options of LEPTOP on theoretical errors. The first two lines indicate parametric uncertainties caused by $\delta s^2 = 0.0003$ (which is equivalent to $\delta\bar{\alpha}^{-1} = 0.12$) and by $\delta m_b = 0.3GeV$. The next six lines refer to the intrinsic theoretical uncertainties. This table was calculated with m_t=175 GeV, M_H=300 GeV and $\hat{\alpha}_s$=0.125.*

	m_W MeV	Γ_l MeV	$\sin^2\theta^l_{eff}$	$\sigma^h_0(nb)$	Γ_Z MeV	Γ_h MeV	R_l	Γ_b MeV	$R_b\times$ 10^5	$R_c\times$ 10^5
δs^2	16	.015	.00031	.0014	.8	.8	.0055	.15	1.1	1.8
δm_b	-	-	-	.0023	.2	.2	.0029	.24	11.0	2.4
$\Delta V_i^{t^2}$	9	.015	.00005	.0002	.5	.4	.0009	.08	.2	.3
$\Delta V_i^{\alpha_s^2}$	5	.008	.00003	.0001	.3	.2	.0005	.04	.1	.2
$\Delta\Gamma_q$	-	-	-	.0029	.3	.3	.0037	-	3.8	1.5
$\Delta\phi^{\alpha_s^2}$	-	-	-	.0010	.1	.1	.0012	.10	4.6	1.0
total	$+13$ -13	$+.023$ $-.023$	$+.00008$ $-.00008$	$+.0032$ $-.0042$	$+1.2$ -1.1	$+1.0$ $-.9$	$+.0063$ $-.0051$	$+.23$ $-.13$	$+8.7$ -4.1	$+1.9$ -2.9

Chapter 3: LEPTOP program

3.1 Introduction

LEPTOP code allows the calculations of LEP-I electroweak observables in the framework of the Minimal Standard Model and fitting the experimental data for determination of m_t, M_H and $\hat{\alpha}_s(M_Z)$. LEPTOP package has been written in the Fortran language and installed on IBM VM and HP-SUN UNIX operating systems. It uses the subroutines from the CERN program library [34]. The code is maintained with the CMZ [35] source code management system. The Fortran code is available either in the form of a compilable source file *leptop.f* or in the CMZ ASCII readable file *leptop.car*. In the following sections we describe simple usage of the LEPTOP package. The user interface part of LEPTOP was written in the object oriented style, which allows a simple use of it without deep knowledge of the Fortran language. The data encapsulation principle and hiding the internal variables are applied. This significantly reduces the amount of information the user should care about improves the robustness of the program.

In order to obtain the maximum accuracy we used whenever possible double-precision variables in the internal calculations. However the user interface was written in a single precision in order to simplify the use of the program, make easy interface with other standard packages and avoid user's bugs typical for the mixture of single and double precision variables. The 7 digit precision (for 32 bit machines) in the input and output is adequate for the present experimental and theoretical errors. The units used in the program are GeV and nanobarns, unless explicitly stated.

This document is a **Reference manual** and **User's Guide**. Whenever possible we tried to follow *cernman* style and notations [36]. An example of a simple program is given in the section 3.8.

This report, including the writeup and Fortran code, is available under WWW [37] on `http://cppm.in2p3.fr/leptop/intro_leptop.html`.

3.2 Initialization

```
CALL LTINIT (ISTAT)
```

Action: The setting of the default values of all variables is done. `LTINIT(0)` should be called before any other `LEPTOP` routine.

Input parameters:

`ISTAT` Level of the initialization.

- `ISTAT=0` means complete initialization from scratch.
- `ISTAT=1` means recalculating constants after the eventual update of the data. This option is needed only for advanced users, because in the user interface it is done automatically.

3.3 Input and output

The output of the program is directed into the unit 6. The unit 19 is reserved for the internal scratch output. The amount of output is controlled via `PRNT` flags (see `LTFLAG` routine).

3.4 Setting of flags and options

```
CALL LTFLAG (KEY,ID)
```

Action: Setting on and off the flags according to the character variable `KEY`, identifying the group of flags, and integer index `ID`, identifying specific flag of that group.

Positive values of `ID` sets the corresponding flag on, negative value of the `ID` sets it off. Zero value of the ID sets off all the flags of the corresponding group. By default all the flags and options are switched off.

Input parameters:

`KEY` Character variable identifying the group of flags.

- `'PRNT'` Group of flags controlling the output level.
- `'OPT'` Group of options modifying the prefered formulas of LEPTOP. See section 2.4 for details.

`ID` Numerical identifier of the flag within the group.
The meaning of different flags is given below:

- `'PRNT'` Allowed values for `ID` are:
  - 1 Low level printout for debug purpose in the process of calculation.
  - 2 Detailed printout during the fit. It may produce very big outputs.
  - 3 Detailed printout after the fit. Printout of the table with MSM and Born predictions for some observables in the style of the table in the paper [9].
  - 4 Printout during the initialization of several default constants and experimental inputs.
  - 8 Printout during the setting of the variables and experimental data is done by the user interface routines `LTPUT, LTFPUT, LTFCOR, LTFUSE` etc.
  - 9 Printout of the calculated observables, setting constants and experimental data is done by the user interface routines `LTGET, LTFGET, LTFIT1, LTFIT2, LTFIT3` etc.
- `'OPT'` Allowed values for `ID` are:
  - 1 Option 1: functions V_A, V_M, V_R are increased by the terms $(2/t)\delta V_i^{t^2}$.
  - 2 Option 2: same variation as option 1, but with negative sign.
  - 3 Option 3: functions V_A, V_M, V_R are increased by the terms $(2/t)\delta V_i^{\alpha_s^2}$.
  - 4 Option 4: same variation as option 3, but with negative sign.

- `5` Option 5: the quark widths of Z are increased in order to take into account estimated QCD corrections to Zqq triangle vertices.
- `6` Option 6: same variation as option 5, but with negative sign.
- `7` Option 7: function $\phi(t)$ is increased by the term $\delta\phi^{\alpha_s^2}$
- `8` Option 8: the b-quark width of Z is increased by 7 MeV.

`'MNUNIT'` Changing the Fortran output unit of MINUIT to unit `ID`. The default is the the output on the scratch file on unit 19. Allowed values for `ID` are from 1 to 99. To restore the output on the scratch file one should assign `ID` to `-19`.

`'MNSAVE'` Changing the SAVE output unit of MINUIT to unit `ID`. The default unit is 7.

`'MNREAD'` Changing the Fortran input unit of MINUIT to unit `ID`. The default value of `ID` is `-5`, which means that MINUIT is working in Fortran-callable mode. **Attention:** using this flag switch MINUIT to the data-driven mode, when user should provide on unit `ID` the complete set of MINUIT commands. If needed, the user can provide his own version of the function FCNLB.

`'MNPRNT'` sets the print level of MINUIT. The default value of `ID` is `-1`, which means no output of MINUIT except from MINUIT `SHOW` command. Other possibles values of `ID` are in the range from 0 to 3 according to MINUIT `SET PRIntout` command.

`'MNEPS'` informs MINUIT that the relative floating point arithmetic precision is 10^{ID}. The default precision is 10^{-10}.

Remark:

`LTFLAG` is the only routine that can be called before `LTINIT(0)`. This allows to suppress the printout during the initialization stage.

Examples:

```
 CALL LTFLAG('OPT',7)
```

This call sets on the option 7.

```
CALL LTFLAG('PRNT',-4)
```

This call sets off the printout of the fit results, if it has been set on previously.

3.5 Setting the constants and experimental data

CALL LTPUT (NAME,DATA)

Action: Setting the constant defined by keyword `NAME` to the value defined by the variable `DATA`.

Input parameters:

`NAME` Character variable containing the name of the constant to be modified.

- `'MT'` Mass of the top quark m_t.
- `'MH'` Mass of the Higgs boson M_H.
- `'MZ'` Mass of the Z boson M_Z.
- `'ALBAR'` Electromagnetic coupling constant $\bar{\alpha}(M_Z)$.
- `'ALSHAT'` Strong coupling constant $\hat{\alpha}_s(M_Z)$.
- `'GFERMI'` Weak Fermi coupling constant G_μ determined from the muon lifetime measurements.

`DATA` New value of the constant.

Remark:

If `PRNT` flag number 8 was set before the call to `LTPUT` the new value will be printed.

Example:

```
CALL LTPUT('MH',300.)
```

This call will set the Higgs mass to 300 GeV.

CALL LTFPUT (TYPE,NAME,DATA)

Action: Setting the values of experimental data, including observables and their errors, for the use in the fit of experimental data with LEPTOP formulas.

Input parameters:

`TYPE` Character variable selecting the type of data to be set.

- `'VALUE'` The central value of the experimental data is set.
- `'ERROR'` The error on the experimental data is set.
- `'MARSEILLE'` Experimental data presented at Marseille-93 conference are loaded in case `NAME='ALL'`.
- `'MORIOND94'` Same, but for Moriond-94 conference.
- `'GLASGOW'` Same, but for Glasgow-94 conference.
- `'MORIOND95'` Same, but for Moriond-95 conference.
- `'BEIJING95'` Same, but for Beijing-95 conference.

`NAME` Character variable identifying the experimental observable. The following observables may be used in the fit:

- `'GZ'` Total width of Z boson Γ_Z.
- `'SIGH'` The hadronic pole cross section $\sigma_h^0 = (12\pi/M_Z^2)(\Gamma_e\Gamma_h/\Gamma_Z^2)$.
- `'RL'` Ratio $R_l = \Gamma_h/\Gamma_l$ of hadronic to leptonic partial width of Z.

`'AFBL'`	Forward-backward lepton asymmetry at Z pole.
`'RB'`	Ratio $R_b = \Gamma_b/\Gamma_h$.
`'MWMZ'`	Ratio M_W/M_Z of W-boson to Z-boson masses.
`'ATAU'`	A_τ from τ polarization.
`'AETAU'`	A_e from τ polarization asymmetry.
`'AFBB'`	Forward-backward b-quark asymmetry at Z pole.
`'AFBC'`	Forward-backward c-quark asymmetry at Z pole.
`'QFB'`	Measurement of forward-backward quark asymmetry translated in a value for $\sin^2\theta_{eff}$.
`'S2NUN'`	Measurement of the ratio of neutral to charged current cross-sections translated into $\sin^2\theta_W$.
`'ALR'`	Left-right polarization asymmetry A_{LR}, measured at SLAC by SLD.
`'MT'`	Top quark mass m_t measured by CDF and D0 at Fermilab.
`'RC'`	Ratio $R_c = \Gamma_c/\Gamma_h$ of charm to hadron partial widths.
`'ALL'`	used to set all observables from the data set defined by the keyword `TYPE`.

`DATA` Numerical value of the experimental data.

Remark:

By default Beijing-95 experimental data are loaded during the initialization with `LTINIT(0)`.

Examples:

```
CALL LTFPUT('VALUE','GZ',2.4974)
```

This call sets the experimental value of Γ_Z to 2.4974 GeV.

```
CALL LTFPUT('ERROR','SIGH',0.12)
```

This call sets the experimental value of the error on the peak hadron cross section to 0.12 nb.

```
CALL LTFPUT('GLASGOW','ALL',dummy)
```

This call sets the input of all experimental data as presented at Glasgow-94 conference.

CALL **LTFCOR** (NAME1,NAME2,DATA)

Action: Set the values of correlation coefficients on the experimental errors, for the use in the fit of experimental data with LEPTOP formulas.

Input parameters:

`NAME1` Character variable identifying the first experimental observable, as defined in the `LTFPUT` description.

`NAME2` Character variable identifying the second experimental observable, as defined in the `LTFPUT` description.

`DATA` Numerical value of the correlation coefficient between the two experimental data.

Example:

```
CALL LTFCOR('GZ','SIGH',-0.110)
```

This call set the correlation coefficient between the experimental errors of Γ_Z and σ_h^0 equal to $\rho_{\Gamma_Z \sigma_h^0} = -0.11$.

3.6 Access to the constants, observables and experimental data

CALL **LTGET** (NAME,DATA*)

Action: Extracting the constant or the theoretical prediction for the observable defined by keyword `NAME` to the variable `DATA`.

Input parameters:

`NAME` Character variable containing the name of the constant or theoretical value of the observable to be extracted.

'MT'	Mass of the top quark m_t.
'MH'	Mass of the Higgs boson M_H.
'MZ'	Mass of the Z boson M_Z.
'ALBAR'	Electromagnetic coupling constant $\bar{\alpha}(M_Z)$.
'ALSHAT'	Strong coupling constant $\hat{\alpha}_s(M_Z)$.
'GFERMI'	Weak Fermi coupling constant G_μ defined by muon lifetime measurements.
'GV'	Vector coupling of the electron.
'GA'	Vector axial coupling of the electron.
'SIN2E'	The effective electro-weak mixing angle $\sin^2\theta_{eff}$ for leptons.
'QFB'	The same as above.
'SIN2B'	The effective electro-weak mixing angle $\sin^2\theta_{eff}$ for b quarks.
'MW'	Mass of the W boson M_W.
'MWMZ'	Ratio of the W boson M_W to Z boson masses.
'GNU'	Partial width of Z boson into $\nu\bar{\nu}$.
'GE'	Partial width Γ_e of Z into e^+e^-.
'GMUON'	Partial width Γ_μ of Z into $\mu^+\mu^-$.
'GTAU'	Partial width Γ_τ of Z into $\tau^+\tau^-$.
'GU'	Partial width Γ_u of Z into $u\bar{u}$ quarks.
'GD'	Partial width Γ_d of Z into $d\bar{d}$ quarks.
'GS'	Partial width Γ_s of Z into $s\bar{s}$ quarks.
'GC'	Partial width Γ_c of Z into $c\bar{c}$ quarks.
'GB'	Partial width Γ_b of Z into $b\bar{b}$ quarks.
'GINV'	Invisible width of Z boson Γ_{inv}.

'GH'	Hadron width of Z boson Γ_h.
'GZ'	Total width of Z boson Γ_Z.
'RL'	Ratio $R_l = \Gamma_h/\Gamma_l$ of hadronic to leptonic partial width of Z.
'RC'	Ratio $R_c = \Gamma_c/\Gamma_h$ of charm to hadron partial widths.
'RB'	Ratio $R_b = \Gamma_b/\Gamma_h$.
'ATAU'	τ-lepton asymmetry A_τ from τ polarization.
'AETAU'	Electron asymmetry A_e from τ polarization.
'AFBL'	Forward backward lepton asymmetry A^l_{FB}.
'AFBC'	Forward-backward c-quark asymmetry at Z pole.
'AFBB'	Forward-backward b-quark asymmetry at Z pole.
'ALR'	Left right asymmetry A_{LR}.
'SIGH'	The hadronic pole cross section $\sigma^0_h = (12\pi/M^2_Z)(\Gamma_e\Gamma_h/\Gamma^2_Z)$.

Output parameter:

`DATA` numerical value of the constant or the result of the calculation for the prediction.

Remark:

- The extracted value may depend on the change of the constants by the previous call to `LTPUT`. So to get a consistent set of observables one should make all calls to `LTGET` after the last call of `LTPUT`.
- If `PRNT` flag number 9 was set before the call to `LTGET` the value of the observable will be printed.

Example:

```
CALL LTGET('MW',AMW)
```

After this call the variable AMW will contain the W boson mass in GeV.

CALL **LTFGET** (TYPE,NAME,DATA*)

Action: Extracting the values of experimental data, including observables and their errors. It also can extract the values of the fitted parameters and their errors after the fit.

Input parameters:

`TYPE` Character variable selecting the type of data to be extracted.

'VALUE'	The central value of the experimental data is given.
'ERROR'	The error on the experimental data is given.
'FIT_RESULT'	The central value of the fitted parameter after the last call to `LTFIT1`, `LTFIT2` or `LTFIT3`.
'FIT_ERROR'	The corresponding parabolic error of the fitted parameter.
'FIT_ERR+'	The corresponding positive error of the fitted parameter.
'FIT_ERR-'	The corresponding negative error.

'FIT_GLB' The corresponding global correlation coefficient with other fitted parameter.

`NAME` Character variable identifying the experimental observable as defined in the description of `LTFPUT` for `TYPE='VALUE'` or `TYPE='ERROR'`. If `NAME='ALL'` and `PRNT` flag 9 is set on, the table of all experimental data is printed. If `TYPE='FIT_*'` the allowed values of `NAME` are: `MT`, `MH` and `ALSHAT`, corresponding to the fitted parameters.

Output parameters:

`DATA` Numerical value of the experimental data or fitted parameter.

Remark:

- If `PRNT` flag number 9 was set before the call to `LTFGET` the value of the observable will be printed.

Examples:

```
CALL LTFGET('VALUE','MWMZ',RMWMZ)
```

After this call the variable RMWMZ will contain the experimental ratio of W boson to Z boson masses M_w/M_z.

```
CALL LTFGET('ERROR','SIGH',ESIGH)
```

After this call the variable ESIGH will contain the experimental value of the error on the peak hadron cross section σ_h^0 in nb.

```
CALL LTFLAG('PRNT',9)

CALL LTFGET(' ','ALL',DUMMY)
```

After these calls the table with all experimental data is printed.

```
CALL LTFIT1('MH',amh,emh,chi2)
CALL LTFGET('FIT_ERR+','MH',emhpos)
CALL LTFGET('FIT_ERR-','MH',emhneg)
```

After these calls the variables `emhpos` and `emhneg` will contain the asymmetric errors on Higgs mass, which might be significantly different from the parabolic error `emh` due to the non-gaussian distributions.

3.7 Fitting experimental data with LEPTOP formulas

The fitting in LEPTOP is done using MINUIT package [38]. The fitted parameter values correspond to the minimum of the χ^2 function:

$$\chi^2 = \sum_{i,j=1}^{n} \frac{(E_i - T_i(m_t, M_H, \hat{\alpha}_s))(E_j - T_j(m_t, M_H, \hat{\alpha}_s))}{S_{ij}} \tag{3.1}$$

$$\frac{1}{S_{ij}} = (V_{ij})^{-1} \tag{3.2}$$

$$V_{ij} = \rho_{ij}\delta E_i \delta E_j \tag{3.3}$$

where the following notations were used:

n	Number of experimental observables used in the fit.
E_i	Experimental observable, which usually comes from the compilation of results of all LEP detectors.
T_i	Theoretical prediction of LEPTOP as a function of parameters m_t, M_H and $\hat{\alpha}_s$.
$V_{i,j}$	Covariance matrix of errors.
δE_i	The experimental error on the observable i.
ρ_{ij}	The correlation coefficient between the experimental errors of the observable i and observable j.

The initial values of parameters are used from the settings done by the routine `LTPUT`. So the user can check the sensitivity of the fit to the initial values of the parameters. It is the user's responsibility to provide reasonable values of the initial parameters, as the minimization procedure may diverge and move the parameters to the unphysical region, where the program might be not protected.

`CALL` **LTFUSE** `(COMMAND,NAME)`

Action: Instructing LEPTOP to use or not to use the particular experimental data in the fit. By default all experimental data listed in `LTFPUT` description are used, except `'MT'`.

Input parameters:

`COMMAND` Character variable selecting the command.

- `'USE'` Command to use this data in the fit.
- `'NOUSE'` Command to exclude this data from the fit.

`NAME` Character variable identifying the experimental observable as defined in the description of `LTFPUT`. If `NAME='ALL'` all experimental data are affected.

Examples:

```
CALL LTFUSE('USE','MT')
```

Add the experimental value of m_t from CDF to the fit.

```
CALL LTFUSE('NOUSE','QFB')
```

Do not use the experimental value of Q_{FB} in the fit.

CALL LTFIT1 (PARAM,PAR1*,EPAR1*,CHI2*)

Action:

Performing one parameter fit of the experimental data. The actual parameter is selected by the character constant `PARAM` from m_t, M_H and $\hat{\alpha}_s$.

Input parameters:

`PARAM` Character variable selecting the parameters.

- `'MT'` Select the parameter m_t.
- `'MH'` Select the parameter M_H.
- `'ALS'` Select the parameter $\hat{\alpha}_s$.

Output parameters:

`PAR1` Value of the fitted parameter.

`EPAR1` Estimate of the error of the fitted parameter.

`CHI2` Value of the χ^2 of the fit.

Example:

```
CALL LTFIT1('MH',AMH,EMH,CHI2)
```

After this call the variable AMH contains the fitted Higgs mass M_H, the variable EMH contains the parabolic estimate of the error of the Higgs boson mass and CHI2 the χ^2 of the fit.

CALL LTFIT2 (PARAM,PAR1*,PAR2*,EPAR1*,EPAR2*,R12*,CHI2*)

Action:

Performing two parameter fit of the experimental data. The actual parameters are selected by the character constant `PARAM` from m_t, M_H and $\hat{\alpha}_s$.

Input parameters:

`PARAM` Character variable selecting the parameters.

- `'MT,ALS'` Select the parameters m_t and $\hat{\alpha}_s$.
- `'MT,MH'` Select the parameters m_t and M_H.
- `'ALS,MH'` Select the parameters M_H and $\hat{\alpha}_s$.

Output parameters:

`PAR1` Value of the first parameter.

`PAR2` Value of the second parameter.

`EPAR1` Value of the error of the first parameter.

`EPAR2` Value of the error of the second parameter.

`R12` Value of the correlation coefficient between two parameters.

`CHI2` Value of the χ^2 of the fit.

Example:

```
CALL LTFIT2('MT,ALS',AMT,ALS,EMT,EALS,R12,CHI2)
```

After this call the variable AMT contains the fitted top quark mass m_t, the variable ALS contains the fitted strong coupling constant $\hat{\alpha}_s(M_Z)$, the variable EMT contains the error of the top quark mass, the variable EALS contains the error of the strong coupling constant, the variable R12 contains the correlation coefficient between the errors of two parameters and CHI2 contains the χ^2 of the fit.

CALL LTFIT3 (PARAM,PAR1*,PAR2*,PAR3*,EPAR1*,EPAR2*,EPAR3*,COV*,CHI2*)

Action:

Performing three parameter fit of the experimental data. The actual parameters are selected by the character constant `PARAM`, actually only m_t, $\hat{\alpha}_s$ and M_H.

Input parameters:

PARAM	Character variable selecting the parameters.
	'MT,ALS,MH'Select the parameters m_t, $\hat{\alpha}_s$ and M_H.

Output parameters:

PAR1	Value of the first parameter.
PAR2	Value of the second parameter.
PAR3	Value of the third parameter.
EPAR1	Value of the error of the first parameter.
EPAR2	Value of the error of the second parameter.
EPAR3	Value of the error of the third parameter.
COV	Covariance matrix of 3 fitted parameters. It should be declared as DIMENSION COV(3,3).
CHI2	Value of the χ^2 of the fit.

Example:

```
CALL LTFIT3('MT,ALS,MH',AMT,ALS,AMH,EMT,EALS,EMH,COV,CHI2)
```

After this call the variable AMT contains the fitted top quark mass m_t, the variable ALS contains the fitted strong coupling constant $\hat{\alpha}_s(M_Z)$, the variable AMH contains the fitted value of the Higgs mass, the variable EMT contains the error of the top quark mass, the variable EALS contains the error of the strong coupling constant, the variable EMH contains the error of the Higgs mass, the array COV(3,3) contains the covariance matrix of the errors of three parameters and CHI2 contains the χ^2 of the fit.

3.8 Sample program

```
program example
```

```
      call ltdemo
      end

      SUBROUTINE LTDEMO
*----- template program to demonstrate LEPTOP package                *

*----- set all LEPTOP print flags off
      CALL LTFLAG('PRNT',0)

*----- initialisation
      CALL LTINIT(0)

*----- get some constants
      CALL LTGET('MZ',AMZ)
      CALL LTGET('GFERMI',GFERMI)
      CALL LTGET('ALBAR',ALBAR)

*----- get more constants
      CALL LTGET('MELE',amele)
      CALL LTGET('MMUO',ammuo)
      CALL LTGET('MTAU',amtau)
      CALL LTGET('MS',amstr)
      CALL LTGET('MC',amchrm)
      CALL LTGET('MB',ambot)

*----- modify some constants

      CALL LTPUT('MT',175.)
      CALL LTPUT('MH',300.)
      CALL LTPUT('ALSHAT',0.125)

*----- get some physical quantities
      CALL LTGET('GV',gv)
      CALL LTGET('GA',ga)
      CALL LTGET('MW',amw)
      CALL LTGET('GNU',gnu)
      CALL LTGET('GE',ge)
      CALL LTGET('GMUON',gmuon)
      CALL LTGET('GTAU',gtau)

*----- fit of mtop and alsbar
      CALL LTFIT2('MT,ALS',amt1,als1,eamt1,eals1,rho1,chi21)

*----- set data from Glasgow conference
      CALL LTFPUT('GLASGOW','ALL',0.)
```

```
*----- modify value of Z total width
      CALL LTFPUT('VALUE','GZ',2.4974)

*----- fit of mtop and mh
      CALL LTFIT2('MT,MH',amt2,amh1,eamt2,eamh1,rho2,chi22)

*----- fit  mt
      CALL LTFIT1('MT',amt3,eamt3,chi23)

*-----  fit  mh
      CALL LTFIT1('MH',amh4,eamh4,chi24)

*----- fit  als
      CALL LTFIT1('ALS',als,eals,chi2)

      END          ! LTDEMO
```

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APPENDIX A

FLOWCHART OF LEPTOP

Select the three most accurate observables:
$G_\mu, M_Z, \alpha(M_Z) \equiv \bar{\alpha}$

Define angle θ ($s \equiv \sin\theta, c \equiv \cos\theta$)
in terms of G_μ, M_Z and $\bar{\alpha}$:
$G_\mu = (\pi/\sqrt{2})\bar{\alpha}/s^2c^2M_Z^2$

Define the Born approximation for other electroweak
observables in terms of G_μ, M_Z and θ.

Introduce bare couplings $(\alpha_0, \alpha_{Z_0}, \alpha_{W_0})$, masses $(M_{Z_0}, M_{W_0}$,
m_{q_0} (including m_{t_0})) and VEV η in the framework of MSM.

Express $\alpha_0, \alpha_{Z_0}, M_{Z_0}$ in terms of G_μ, M_Z and $\bar{\alpha}$ in the one-loop
approximation, using dimensional regularization $(1/\epsilon, \mu)$

Express one-loop corrections to all other electroweak observables
in terms of $\alpha_0, \alpha_{Z_0}, M_{Z_0}, m_t, M_H$ and hence in terms of G_μ,
$M_Z, \bar{\alpha}, m_t, M_H$. Check cancelation of $1/\epsilon$ and μ.

Introduce gluonic corrections into quark loops and QED and QCD
final state interactions for hadronic decays
(in terms of $\bar{\alpha}, \hat{\alpha}_s(m_Z)^*, m_b(M_Z), m_t$).

Compare the Born results and Born + one loop results with
experimental data on Z - decays and M_W.

Make a global fit for three parameters $m_t, M_H, \hat{\alpha}_s(M_Z)$.

Predict the central values of all electroweak observables
and their uncertainties.

$*$ – $\hat{\alpha}_s(M_Z)$ is the QCD coupling in $\overline{\mathrm{MS}}$ - scheme.

APPENDIX B

List of LEPTOP papers [1]

1. Electroweak radiative corrections and top quark mass, CERN-TH.6053/91, March 1991, TPI-MINN-91/14-I, ITEP-15/91.

2. Parametrization of electroweak radiative corrections, ITEP-104/92, November 1992; *ZhETF* **103** (1993) 1489 (in Russian), May 1993, *JETP* **76** (1993) 725.

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4. On the interpretation of the CHARM II data, CERN-TH.6695/92, October 1992; *Phys.Lett.* **B298** (1993) 453.

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