Rep. Prog. Phys. **62** (1999) 1–59. Printed in the UK PII: S0034-4885(99)14976-5

Theory of Z boson decays

V A Novikov†, L B Okun†, A N Rozanov‡ and M I Vysotsky§
† ITEP, 117218 Moscow, Russia
‡ ITEP and CPPM, CNRS/IN2P3 - Univ. Mediterranée, Marseille, France
§ ITEP and INFN, Sezione di Ferrara, Italy

E-mail: novikov@heron.itep.ru; okun@heron.itep.ru; vysotsky@heron.itep.ru and rozanov@cppm.in2p3.fr

Received 18 March 1999

Abstract

The precision data on Z boson decays from LEP-I and SLC colliders are compared with the predictions based on the minimal standard theory. The Born approximation of the theory is based on three most accurately known observables: G_μ—the four fermion coupling constant of muon decay, m_Z—the mass of the Z boson, and $\alpha(m_Z)$—the value of the 'running fine structure constant' at the scale of m_Z. The electroweak loop corrections are expressed, in addition, in terms of the masses of higgs, m_H, of the top and bottom quarks, m_t and m_b, and of the strong interaction constant $\alpha_s(m_Z)$. The main emphasis of the review is focused on the one-electroweak-loop approximation. Two electroweak loops have been calculated in the literature only partly. Possible manifestations of new physics are briefly discussed.

Contents

		Page
1.	Introduction	3
2.	Basic parameters of the theory	8
3.	Amplitudes, widths and asymmetries	11
4.	One-loop corrections to hadronless observables	13
	4.1. Four types of Feynman diagrams	13
	4.2. The asymptotic limit at $m_t^2 \gg m_Z^2$	15
	4.3. The functions $V_m(t,h)$, $V_A(t,h)$ and $V_R(t,h)$	15
	4.4. Corrections $\delta V_i(t)$	17
	4.5. Accidental (?) compensation and the mass of the t quark	19
	4.6. How to calculate V_i? ‘Five steps’	21
5.	One-loop corrections to hadronic decays of the Z boson	24
	5.1. The leading quarks and hadrons	24
	5.2. Decays to pairs of light quarks	24
	5.3. Decays to $b\bar{b}$ pair	25
6.	Comparison of one-electroweak-loop results and experimental LEP-I and SLC data	26
	6.1. `LEPTOP` code	26
	6.2. One-loop general fit	26
7.	Two-loop electroweak corrections and theoretical uncertainties	28
	7.1. $\alpha_W^2 t^2$ corrections to m_W/m_Z, g_A and g_V/g_A from reducible diagrams	28
	7.2. $\alpha_W^2 t^2$ corrections from irreducible diagrams	29
	7.3. $\alpha_W^2 t$ corrections and the two-loop fit of experimental data	31
8.	Extensions of the SM	33
	8.1. Sequential heavy generations in the SM	34
	8.2. SUSY extensions of the SM	36
9.	Conclusions	41
Appendix A.	Regularization of Feynman integrals	43
Appendix B.	Relation between $\bar{\alpha}$ and $\alpha(0)$	44
Appendix C.	How $\alpha_W(q^2)$ and $\alpha_Z(q^2)$ ‘crawl’	47
Appendix D.	General expressions for one-loop corrections to hadronless observables	48
Appendix E.	Radiators R_{Aq} and R_{Vq}	50
Appendix F.	$\alpha_W^2 t^2$ corrections from reducible diagrams	51
Appendix G.	Oblique corrections from new generations and SUSY	53
Appendix H.	Other parametrizations of radiative corrections	54

1. Introduction

The Z boson, the electrically neutral vector boson (its spin equals one) with mass $m_Z \simeq 91$ GeV and width $\Gamma_Z \simeq 2.5$ GeV†, occupies a unique place in physics. This heavy analogue of the photon was experimentally discovered in 1983, practically simultaneously with its charged counterparts, the $W^{\pm}$ bosons with mass $m_W \simeq 80$ GeV and width $\Gamma_W \simeq 2$ GeV [1].

The discovery was crowned with Nobel prizes to Carlo Rubbia (for the bosons) and to Simon van der Meer (for the CERN proton–antiproton collider, which was specially constructed to produce W and Z bosons) [2].

The extremely short-lived vector bosons ($\tau = 1/\Gamma \simeq 10^{-25}$ s) were detected by their decays into various leptons and hadrons. The detectors in which these decay products were observed, were built and operated by collaborations of physicists and engineers: the largest in the history of physics.

The discovery of W and Z bosons was a great triumph of experimental physics, but even more so of theoretical physics. The masses and widths of the particles, the cross sections of their production turned out to be in perfect agreement with the predictions of electroweak theory of Sheldon Glashow, Abdus Salam and Steven Weinberg [3]. The theory was so beautiful that its authors received the Nobel prize in 1979 [4], four years before its crucial confirmation.

The electroweak theory unified two types of fundamental interactions: electromagnetic and weak. The theory of electromagnetic interaction—quantum electrodynamics (QED)—was cast in its present relativistically covariant form in the late 1940s and early 1950s and served as a ‘role model’ for the relativistic field theories of two other fundamental interactions: weak and strong.

The main virtue of QED was its renormalizability. Let us explain this ‘technical’ term by using the example of interaction of photons with electrons. One can find a systematic presentation in modern textbooks [5]. In the lowest approximation of perturbation theory (the so-called tree approximation in the language of Feynman diagrams) all electromagnetic phenomena can be described in terms of electric charge and mass of the electron (e, m). The small parameter of perturbation theory is the well known $\alpha = e^2/4\pi \simeq \frac{1}{137}$.

The problem with any quantum field theory is that in higher orders of perturbation theory, described by Feynman graphs with loops, the integrals over momenta of virtual particles have ultraviolet divergences, so that all physical quantities including the electric charge and mass of the electron themselves become infinitely large. To avoid infinities an ultraviolet cut-off Λ could be introduced. Another, more sophisticated method is to use dimensional regularization: to calculate the Feynman integrals in momentum space of D dimensions. These integrals diverge at $D = 4$, but are finite, proportional to $1/\varepsilon$ in the vicinity of $D = 4$, where by definition $2\varepsilon = 4 - D \to 0$ (see appendix A).

The theory is called renormalizable if one can get rid of this cut-off (or $1/\varepsilon$) by establishing relations between observables only. In the case of electrons and photons, such basic observables are physical (renormalized) charge and mass of the electron. This allows one to calculate higher-order effects in α and compare the theoretical predictions with the results of high-precision measurements of such observables as, e.g., anomalous magnetic moments of electron or muon.

The renormalizability of electrodynamics is guaranteed by the dimensionless nature of the coupling constant α and by conservation of electric current.

After this short description of QED let us turn to the weak interaction.

The first manifestation of the weak interaction was discovered by Henri Becquerel at the

† Throughout the paper we use units in which $\hbar, c = 1$.

end of the 19th century. Later, this type of radioactivity was called β-decay. The first theory of β-decay was proposed by Enrico Fermi in 1934 [6]. The theory was modelled after quantum electrodynamics with two major differences: first, instead of charge conserving, 'neutral', electrical current of the type $-\bar{e}\gamma_\alpha e + \bar{p}\gamma_\alpha p$ there were introduced two charge changing, 'charged', vector currents: one for nucleons, transforming neutron into a proton, $\bar{p}\gamma_\alpha n$, another for leptons, transforming neutrino into electron or creating a pair: electron plus antineutrino, $\bar{e}\gamma_\alpha \nu$. (Here $\bar{e}(e)$ denotes operator, which creates (annihilates) electron and annihilates (creates) positron. The symbols of other particles have similar meaning; γ_α are four Dirac matrices, $\alpha = 0, 1, 2, 3$.)

The second difference between the Fermi theory and electrodynamics was that the charged currents interacted locally via four-fermion interaction:

$$G \cdot \bar{p}\gamma_\alpha n \cdot \bar{e}\gamma_\alpha \nu + \text{h.c.}, \tag{1}$$

where summation over index α is implied (in this summation we use Feynman's convention: + for $\alpha = 0$ and $-$ for $\alpha = 1, 2, 3$); h.c. stands for Hermitian conjugate. The coupling constant G of this interaction is called the Fermi coupling constant.

From simple dimensional considerations it is evident that the dimension of G is $(\text{mass})^{-2}$ and therefore the four-fermion interaction is not renormalizable, the higher orders being divergent as $G^2\Lambda^2$, $G^3\Lambda^4, \ldots$. Why these divergent corrections still allow one to rely on the lowest order approximation remained a mystery. But for many years the lowest order four-fermion interaction served as a successful phenomenological theory of weak interactions.

It is in the framework of this phenomenological theory that a number of subsequent experimental discoveries were accommodated. First, it turned out that β-decay is one of the large family of weak processes, involving newly discovered particles, such as pions, muons and muonic neutrinos, strange particles, etc. Second, it was discovered that all these processes are caused by the self-interaction of one weak charged current, involving leptonic and hadronic terms. Later on, when the quark structure of hadrons was established, the hadronic part of the current was expressed through the corresponding quark current. Third, it was established in 1957 that all weak interactions violate parity conservation P and charge conjugation invariance C. This violation turned out to have a universal pattern: the vector form of the current V, introduced by Fermi, was substituted [7] by one-half of the sum of vector and axial vector, A, which meant that γ_α should be substituted by $\frac{1}{2}\gamma_\alpha(1 + \gamma_5)$.

In other words, one can say that fermion ψ enters the charged current only through its left-handed chiral component

$$\psi_L = \tfrac{1}{2}(1 + \gamma_5)\psi. \tag{2}$$

From such a structure of the charged current it follows that the corresponding antifermions interact only through their right-handed components.

Attempts to construct a renormalizable theory of weak interaction resulted in a unified theory of electromagnetic and weak interactions—the electroweak theory [3,4], with two major predictions. The first prediction was the existence alongside the charged weak current of a neutral weak current. The second prediction was the existence of the vector bosons: Z coupled to the weak neutral current and W^+ and W^- coupled to the charged current (as $\frac{1}{2}\bar{e}\gamma_\alpha(1 + \gamma_5)\nu$) and its Hermitian conjugate current (as $\frac{1}{2}\bar{\nu}\gamma_\alpha(1 + \gamma_5)e$).

The vector bosons were massive analogues of the photon γ; their couplings to the corresponding currents, f and g, were the analogues of the electric charge e. Thus $\alpha_Z = f^2/4\pi$ and $\alpha_W = g^2/4\pi$ were dimensionless like $\alpha = e^2/4\pi$, which was a necessary (but not sufficient) condition of renormalizability of the weak interaction.

The first theory, involving charged vector bosons and photon, was proposed by Oscar Klein just before World War II [8]. Klein based his theory on the notion of local isotopic symmetry:

he considered isotopic doublets (p, n) and (ν, e), and the isotopic triplet (B^+, A^0, B^-). $B^\pm$ denoted what we now call $W^\pm$, while A^0 was the electromagnetic field. He also mentioned the possibility of incorporating a neutral massive field C^0 (the analogue of our Z^0). In fact this was the first attempt to construct a theory based on a non-Abelian gauge symmetry, with vector fields playing the role of gauge fields. The gauge symmetry was essential for conservation of the currents. Unfortunately, Klein did not discriminate between weak and strong interaction and his paper was firmly forgotten.

The non-Abelian gauge theory was rediscovered in 1954 by C N Yang and R Mills [9] and became the basis of the so-called standard model (SM) with its colour $SU(3)_c$ group for strong interaction of quarks and gluons and $SU(2)_L \times U(1)_Y$ group for electroweak interaction (here indices denote: c, colour, L, weak isospin of left-handed spinors and Y, the weak hypercharge). The electric charge $Q = T_3 + Y/2$, where T_3 is the third projection of isospin. $Y = \frac{1}{3}$ for a doublet of quarks, $Y = -1$ for a doublet of leptons. As for the right-handed spinors, they are isosinglets, and hence

$$Y(\nu_R) = 0, \qquad Y(\mathrm{e}_R) = -2, \qquad Y(\mathrm{u}_R) = \tfrac{4}{3}, \qquad Y(\mathrm{d}_R) = -\tfrac{2}{3}.$$

Thus, parity violation and charge conjugation violation were incorporated into the foundation of electroweak theory.

Out of the four fields (three of $SU(2)$ and one of $U(1)$, usually denoted by W^+, W^0, W^- and B^0, respectively) only two directly correspond to the observed vector bosons: W^+ and W^-. The Z^0 boson and photon are represented by two orthogonal superpositions of W^0 and B^0:

$$\begin{aligned} Z^0 &= cW^0 - sB^0 \\ A^0 &= sW^0 + cB^0, \end{aligned} \tag{3}$$

where $c = \cos\theta$, $s = \sin\theta$, while the weak angle θ is a free parameter of electroweak theory. The value of θ is determined from experimental data on Z boson coupling to neutral current. The 'Z-charge', characterizing the coupling of the Z boson to a spinor with definite helicity is given by

$$\bar{f}(T_3 - Qs^2), \tag{4}$$

where†

$$\bar{f} = \bar{g}/c. \tag{5}$$

Note that the 'Z-charge' is different for the right- and left-handed spinors with the same value of Q because they have different values of T_3. The coupling constant of W bosons is also expressed in terms of $\bar{e}$ and θ:

$$\bar{g} = \bar{e}/s. \tag{6}$$

The theory described above has many nice features, the most important of which is its renormalizability. But at first sight it looks absolutely useless: all fermions and bosons in it are massless. This drawback cannot be fixed by simply adding mass terms to the Lagrangian. The mass terms of fermions would contain both ψ_L and ψ_R and thus explicitly break the isotopic invariance and hence renormalizability. The gauge invariance would also be broken by the mass terms of the vector bosons. All this would result in divergences of the type Λ^2/m^2, Λ^4/m^4, etc.

The way out of this trap is the so-called Higgs mechanism [10]. In the framework of the minimal standard model (MSM) the problem of mass is solved by postulating the existence

† We denote by $\bar{e}$, $\bar{f}$, $\bar{g}$ the values of the corresponding charges at m_Z scale, while e, f, g refer to values at vanishing momentum transfer. The same applies to $\bar{\alpha}$, $\bar{\alpha}_Z$, $\bar{\alpha}_W$ and α, α_Z, α_W (see equations (13)–(18)).

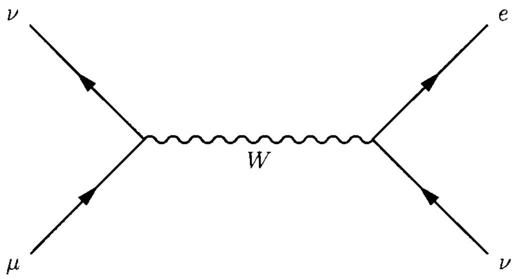

Figure 1. Muon decay in the tree approximation.

of the doublet $\varphi_H = (\varphi^+, \varphi^0)$ and corresponding antidoublet $(\bar{\varphi}^0, -\varphi^-)$ of spinless particles. These four bosons differ from all other particles by the form of their self-interaction, the energy of which is minimal when the neutral field $\varphi_1 = \frac{1}{\sqrt{2}}(\varphi^0 + \bar{\varphi}^0)$ has a nonvanishing vacuum expectation value. The isospin of the Higgs doublet is $\frac{1}{2}$, its hypercharge is 1. Thus, it interacts with all four gauge bosons. In particular, it has quartic terms $\frac{1}{4}\bar{g}^2\bar{W}W\varphi_1^2$, $\frac{1}{8}\bar{f}^2\bar{Z}Z\varphi_1^2$, which give masses to the vector bosons when φ_1 acquires its vacuum expectation value (VEV) η:

$$m_W = \bar{g}\eta/2, \qquad m_Z = \bar{f}\eta/2. \tag{7}$$

The magnitude of η can easily be derived from that of the four-fermion interaction constant G_μ in muon decay:

$$\frac{G_\mu}{\sqrt{2}} \cdot \bar{\nu}_\mu\gamma_\alpha(1+\gamma_5)\mu \cdot \bar{e}\gamma_\alpha(1+\gamma_5)\nu_e. \tag{8}$$

In the Born approximation of electroweak theory this four-fermion interaction is caused by an exchange of a virtual W boson (see figure 1). Hence†

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8m_W^2} = \frac{\bar{g}^2}{8m_W^2}, \qquad \eta = (\sqrt{2}G_\mu)^{-1/2} = 246 \ \text{GeV}. \tag{9}$$

Such mechanism of appearance of masses of W and Z bosons is called spontaneous symmetry breaking. It preserves renormalizability [11]. (As a hint, one can use the symmetrical form of Lagrangian by not specifying the VEV η.)

The fermion masses can be introduced also without explicitly breaking the gauge symmetry. In this case the mass arises from an isotopically invariant term $f_Y \cdot \varphi_H \bar{\psi}_L \psi_R + \text{h.c.}$, where f_Y is called the Yukawa coupling. The mass of a fermion $m = f_Y\eta/\sqrt{2}$. There is a separate Yukawa coupling for each of the known fermions. Their largely varying values are at present free parameters of the theory and await further understanding of this hierarchy.

Let us return for a moment to the vector bosons. A massless vector boson (e.g. photon) has two spin degrees of freedom—two helicity states. A massive vector boson has three spin degrees of freedom corresponding, say to projections $\pm 1, 0$ on its momentum. Under spontaneous symmetry breaking three out of four spinless states, $\varphi^\pm, \varphi_2^0 = \frac{1}{\sqrt{2}}(\varphi^0 - \bar{\varphi}^0)$ become third components of the massive vector bosons. Thus, in the MSM there must exist only one extra particle: a neutral Higgs scalar boson, or simply, higgs‡, representing a quantum of excitation of field φ_1^0 over its VEV η. The discovery of this particle is crucial for testing the correctness of MSM.

† For more on electroweak Born approximation, for which equality $g = \bar{g}$ holds, see equations (14)–(25).

‡ Throughout this review we consistently use capital ‘H’ in such terms as ‘Higgs mechanism’, ‘Higgs boson’, ‘Higgs doublet’, but the lower case of ‘higgs’ is used as a name of the particle, not the man.

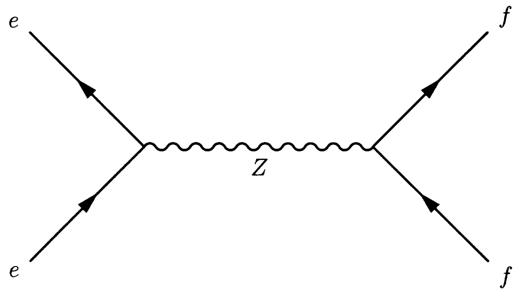

Figure 2. The Z boson as a resonance in e^+e^- annihilations.

The first successful test of electroweak theory was provided by the discovery of neutral currents in the interaction of neutrinos with nucleons [12]. Further study of this deep inelastic scattering (DIS) allowed to extract the rough value of the $\sin\theta$: $s^2 \simeq 0.23$ and thus to predict the values of $m_W \simeq 80$ GeV and $m_Z \simeq 90$ GeV, which served as leading lights for the discovery of these particles.

A few other neutral current interactions have been discovered and studied: neutrino–electron scattering [13], parity violating electron–nucleon scattering at high energies [14] and parity violation in atoms [15]. All of them turned out to be in agreement with electroweak theory. A major part of the theoretical work on electroweak corrections prior to the discovery of the W and Z bosons was devoted to calculating the neutrino–electron [16] (and especially nucleon–electron [17]) interaction cross sections.

After the discovery of the W and Z bosons it became evident that the next level in the study of electroweak physics must consist of precision measurements of production and decays of Z bosons in order to test the electroweak radiative correction. For such measurements, special electron–positron colliders SLC (at SLAC) and LEP-I (at CERN) were constructed and started to operate in the fall of 1989. SLC had one intersection point of colliding beams and hence one detector (SLD); LEP-I had four intersection points and four detectors: ALEPH, DELPHI, L3 and OPAL.

In connection with the construction of LEP and SLC, a number of teams of theorists carried out detailed calculations of the required radiative corrections. These calculations were discussed and compared at special workshops and meetings. The result of this work was the publication of two so-called 'CERN yellow reports' [18, 19], which, together with the yellow report [20], became the 'must' books for experimentalists and theoreticians studying the Z boson. The book [21] (which should be published in 1999) summarizes results of theoretical studies.

More than 2000 experimentalists and engineers and hundreds of theorists participated in this unique collective quest for truth!

The sum of energies of $e^+ + e^-$ was chosen to be equal to the Z boson mass. LEP-I was terminated in the fall of 1995 in order to give place to LEP-II, which will operate in the same tunnel till 2001 with maximal energy 200 GeV. SLC continued at energy close to 91 GeV.

The reactions which have been studied at LEP-I and SLC may be presented in the form (see figure 2):

$$e^+e^- \to Z \to f\bar{f}, \tag{10}$$

where

$$f\bar{f} = \nu\bar{\nu}(\nu_e\bar{\nu}_e,\ \nu_\mu\bar{\nu}_\mu,\ \nu_\tau\bar{\nu}_\tau) \qquad \text{invisible,}$$

$l\bar{l}(e\bar{e}, \mu\bar{\mu}, \tau\bar{\tau})$ charged leptons,
$q\bar{q}(u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}, b\bar{b})$ hadrons.

About 20 000 000 Z bosons have been detected at LEP-I and 550 000 at SLC (but here electrons are polarized, which compensates for the lower number of events).

Experimental data from all five detectors were summarized and analysed by the LEP Electroweak Working Group and the SLD Heavy Flavour and Electroweak Groups which prepared a special report 'A combination of preliminary electroweak measurements and constraints on the standard model' [22]. These data were analysed in [22] by using ZFITTER code (see section 6.1) and independently by J Erler and P Langacker [23].

Fantastic precision has been reached in the measurement of the Z boson mass and width [22]:

$$m_Z = 91\,186.7(2.1)\ \text{MeV}, \qquad \Gamma_Z = 2493.9 \pm 2.4\ \text{MeV}. \tag{11}$$

Of special interest is the measurement of the width of invisible decays of Z:

$$\Gamma_{\text{invisible}} = 500.1 \pm 1.9\text{MeV}. \tag{12}$$

By comparing this number with theoretical predictions for neutrino decays it was established that the number of neutrinos which interact with the Z boson is three ($N_\nu = 2.994 \pm 0.011$). This is a result of fundamental importance. It means that there exist only three standard families (or generations) of leptons and quarks†. Extra families (if they exist) must have either very heavy neutrinos ($m_N > m_Z/2$), or no neutrinos at all.

This review is devoted to the description of the theory of electroweak radiative corrections in Z boson decays and to their comparison with experimental data [22]. Our approach to the theory of electroweak corrections differs somewhat from that used in [18–23]. We believe that it is simpler and more transparent (see section 6.1). In section 2 we introduce the basic input parameters of the electroweak Born approximation. In section 3 we present phenomenological formulae for amplitudes, decay widths, and asymmetries of the numerous decay channels of Z bosons. The main subject of our review is the calculation of one-electroweak loop radiative corrections to the Born approximation. In section 4 they are calculated to the hadronless decays and the mass of the W boson, while in section 5—to the hadronic decays. In section 6 the results of the one-electroweak loop calculations are compared with the experimental data. Section 7 gives a sketch of two-electroweak loop corrections and of their influence on the fit of experimental data. Section 8 discusses possible manifestations of new physics (extra generations of fermions and supersymmetry). Section 9 contains conclusions.

In order to make the reading of the main text easier, technical details and derivations are collected in the appendices.

2. Basic parameters of the theory

The first step in the theoretical analysis is to separate genuinely electroweak effects from purely electromagnetic ones, such as real photons emitted by initial and final particles in reaction (10) and virtual photons emitted and absorbed by them. The electroweak quantities extracted in this way are called sometimes [20] pseudo-observables, but for the sake of brevity we will refer to them as observables.

A key role among purely electromagnetic effects is played by a phenomenon which is called the running of electromagnetic coupling 'constant' $\alpha(q^2)$. The dependence of the electric

† Combining equation (12) with the data on $\nu_\mu e^-$ [24] and $\nu_e e^-$ [25] scattering allowed it to be established that ν_e, ν_μ and ν_τ have equal values of couplings with the Z boson [26].

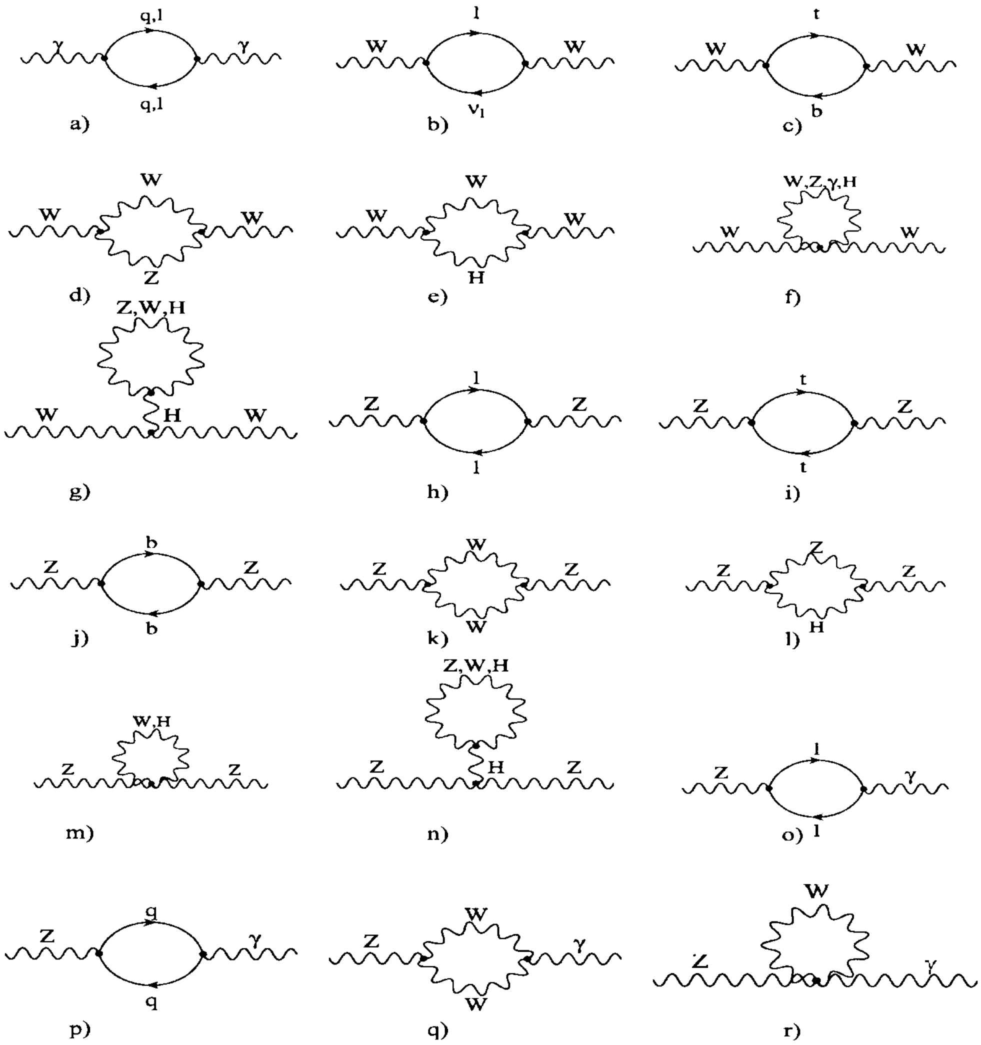

Figure 3. Photon polarization of the vacuum, resulting in the logarithmic running of the electromagnetic charge e and the 'fine structure constant' $\alpha \equiv \frac{e^2}{4\pi}$, as a function of q^2, where q is the 4-momentum of the photon (a). Some of the diagrams that contribute to the self-energy of the W boson (b)–(g). Some of the diagrams that contribute to the self-energy of the Z boson (h)–(n). Some of the diagrams that contribute to the $Z \leftrightarrow \gamma$ transition (o)–(r).

charge on the square of the four-momentum transfer q^2 is caused by the photon polarization of vacuum, i.e. by loops of charged leptons and quarks (hadrons) (see figure 3(a)).

As is well known (see e.g. [27])

$$\alpha \equiv \alpha(q^2 = 0) = [137.035\,985(61)]^{-1}. \tag{13}$$

It has a very high accuracy and is very important in the theory of electromagnetic processes at low energies. As for electroweak processes in general and Z decays in particular, they are determined by [22]

$$\bar{\alpha} \equiv \alpha(q^2 = m_Z^2) = [128.878(90)]^{-1}, \tag{14}$$

the accuracy of which is much worse.

It is convenient to denote

$$\bar{\alpha} = \frac{\alpha}{1 - \delta\alpha}, \tag{15}$$

where

$$\delta\alpha = \delta\alpha_l + \delta\alpha_h = 0.031\,498(0) + 0.059\,40(66) \tag{16}$$

(for the value of $\delta\alpha_l$ see [28], while for the value of $\delta\alpha_h$ see [29]).

It is obvious that the uncertainty of $\delta\alpha$ and hence of $\bar{\alpha}$ stems from that of hadronic contribution $\delta\alpha_h$.

While $\alpha(q^2)$ is running electromagnetically fast, $\alpha_Z(q^2)$ and $\alpha_W(q^2)$ are 'crawling' electroweakly slow for $q^2 \lesssim m_Z^2$:

$$\alpha_Z \equiv \alpha_Z(0) = \tfrac{1}{23.10}, \qquad \bar{\alpha}_Z \equiv \alpha_Z(m_Z^2) = \tfrac{1}{22.91} \tag{17}$$

$$\alpha_W \equiv \alpha_W(0) = \tfrac{1}{29.01}, \qquad \bar{\alpha}_W \equiv \alpha_W(m_Z^2) = \tfrac{1}{28.74}. \tag{18}$$

The small differences $\alpha_Z - \bar{\alpha}_Z$ and $\alpha_W - \bar{\alpha}_W$ are caused by electroweak radiative corrections. Therefore one could and should neglect them when defining the electroweak Born approximation. (We used this recipe ($g = \bar{g}$) when deriving the relation (9) between G_μ and η.)

The theoretical analysis of electroweak effects in this report is based on the three most accurately known parameters: G_μ, $\bar{\alpha}$ (equation (14)) and m_Z (equation (11)).

$$G_\mu = 1.166\,39(1) \times 10^{-5}\ \ \mathrm{GeV}^{-2}. \tag{19}$$

This value of G_μ [27] is extracted from the muon lifetime after taking into account the purely electromagnetic corrections (including bremsstrahlung) and kinematical factors [31]:

$$\frac{1}{\tau_\mu} \equiv \Gamma_\mu = \frac{G_\mu^2 m_\mu^5}{192\pi^3} f\left(\frac{m_e^2}{m_\mu^2}\right)\left[1 - \frac{\alpha(m_\mu)}{2\pi}\left(\pi^2 - \frac{25}{4}\right)\right], \tag{20}$$

where

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x,$$

and

$$\alpha(m_\mu)^{-1} = \alpha^{-1} - \frac{2}{3\pi}\log\left(\frac{m_\mu}{m_e}\right) + \frac{1}{6\pi} \approx 136.$$

Now we are ready to express the weak angle θ in terms of G_μ, $\bar{\alpha}$ and m_Z. Starting from equations (9), (5) and (6), we get in the electroweak Born approximation:

$$G_\mu = \frac{\bar{g}^2}{4\sqrt{2}m_W^2} = \frac{\bar{f}^2}{4\sqrt{2}m_Z^2} = \frac{\pi\bar{\alpha}}{\sqrt{2}m_Z^2 s^2 c^2} \tag{21}$$

from which

$$\begin{aligned} \bar{f}^2 &= 4\sqrt{2}G_\mu m_Z^2 = 0.548\,63(3), \\ \bar{f} &= 0.740\,70(2) \end{aligned} \tag{22}$$

$$\sin^2 2\theta = 4\pi\bar{\alpha}/\sqrt{2}G_\mu m_Z^2 = 0.710\,90(50), \tag{23}$$

$$s^2 = 0.231\,16(23), \tag{24}$$

$$c = 0.876\,83(13). \tag{25}$$

The angle θ was introduced in the mid-1980s [32]. However, its consistent use began only in the 1990s [33]. Using θ automatically takes into account the running of $\alpha(q^2)$ and makes it possible to concentrate on genuinely electroweak corrections as will be demonstrated below.

(In this review we consistently use m_Z as defined by EWWG in accord with our equation (D.7). Note that a different definition of the Z boson mass $\overline{m_Z}$ is known in the literature, related to a different parametrization of the shape of the Z boson peak [34].)

The introduction of the Born approximation described above differs from the traditional approach in which $\bar{\alpha} - \alpha$ is treated as the largest electroweak correction, masses m_W and m_Z are handled on an equal footing, and the angle θ_W, defined by

$$c_W \equiv \cos\theta_W = m_W/m_Z, \qquad s_W^2 = 1 - c_W^2, \tag{26}$$

is considered as one of the basic parameters of the theory. (Note that the experimental accuracy of θ_W is much worse than that of θ.)

After discussing our approach and its main parameters we are prepared to consider various decays of Z bosons.

3. Amplitudes, widths and asymmetries

Phenomenologically, the amplitude of the Z boson decay into a fermion-antifermion pair $f\bar{f}$ can be presented in the form:

$$M(Z \to f\bar{f}) = \tfrac{1}{2}\bar{f}\bar{\psi}_f(g_{Vf}\gamma_\alpha + g_{Af}\gamma_\alpha\gamma_5)\psi_f Z_\alpha, \tag{27}$$

where coefficient $\bar{f}$ is given by equation (22)†. In the case of neutrino decay channel there is no final state interaction or bremsstrahlung. Therefore, the width into any pair of neutrinos is given by

$$\Gamma_\nu \equiv \Gamma(Z \to \nu\bar{\nu}) = 4\Gamma_0(g_{A\nu}^2 + g_{V\nu}^2) = 8\Gamma_0 g_\nu^2, \tag{28}$$

where neutrino masses are assumed to be negligible, and Γ_0 is the so-called standard width:

$$\Gamma_0 = \frac{\bar{f}^2 m_Z}{192\pi} = \frac{G_\mu m_Z^3}{24\sqrt{2}\pi} = 82.940(6)\ \text{MeV}. \tag{29}$$

For decays to any of the pairs of charged leptons $l\bar{l}$ we have:

$$\Gamma_l \equiv \Gamma(Z \to l\bar{l}) = 4\Gamma_0\left[g_{Vl}^2\left(1 + \frac{3\bar{\alpha}}{4\pi}\right) + g_{Al}^2\left(1 + \frac{3\bar{\alpha}}{4\pi} - 6\frac{m_l^2}{m_Z^2}\right)\right]. \tag{30}$$

The QED 'radiator' $(1+3\bar{\alpha}/4\pi)$ is due to bremsstrahlung of real photons and emission and absorption of virtual photons by l and $\bar{l}$. Note that it is expressed not through α, but through $\bar{\alpha}$.

For the decays to any of the five pairs of quarks $q\bar{q}$ we have

$$\Gamma_q \equiv \Gamma(Z \to q\bar{q}) = 12\Gamma_0[g_{Aq}^2 R_{Aq} + g_{Vq}^2 R_{Vq}]. \tag{31}$$

Here an extra factor of three in comparison with leptons takes into account the three colours of each quark. The radiators R_{Aq} and R_{Vq} contain contributions from the final state gluons and photons. In the crudest approximation

$$R_{Vq} = R_{Aq} = 1 + \frac{\hat{\alpha}_s}{\pi}, \tag{32}$$

where $\alpha_s(q^2)$ is the QCD running coupling constant:

$$\hat{\alpha}_s \equiv \alpha_s(q^2 = m_Z^2) \simeq 0.12. \tag{33}$$

(For additional details on $\hat{\alpha}_s$ and radiators see appendix E.)

† Z boson couplings are diagonal in flavour unlike those of the W boson, where the Cabibbo–Kobayashi–Maskawa mixing matrix [35] should be accounted for in the case of couplings with quarks.

The full hadron width (to the accuracy of very small corrections, see sections 5 and 7) is the sum of widths of five quark channels:

$$\Gamma_h = \Gamma_u + \Gamma_d + \Gamma_s + \Gamma_c + \Gamma_b. \quad (34)$$

The total width of the Z boson:

$$\Gamma_Z = \Gamma_h + \Gamma_e + \Gamma_\mu + \Gamma_\tau + 3\Gamma_\nu. \quad (35)$$

The cross section of annihilation of e^+e^- into hadrons at the Z peak is given by the Breit–Wigner formula

$$\sigma_h = \frac{12\pi}{m_Z^2} \frac{\Gamma_e \Gamma_h}{\Gamma_Z^2}. \quad (36)$$

Finally the following notations for the ratio of partial widths are widely used:

$$R_b = \frac{\Gamma_b}{\Gamma_h}, \qquad R_c = \frac{\Gamma_c}{\Gamma_h}, \qquad R_l = \frac{\Gamma_h}{\Gamma_l}. \quad (37)$$

(Note that Γ_l in the numerator of R_l refers to a single charged lepton channel, whose lepton mass is neglected.)

Parity violating interference of g_{Af} and g_{Vf} leads to a number of effects: forward–backward asymmetries A_{FB}, longitudinal polarization of τ-lepton P_τ, dependence of the total cross section at Z peak on the longitudinal polarization of the initial electron beam A_{LR}, etc. Let us define for the channels of charged lepton and light quark (u, d, s, c) whose mass may be neglected the quantity

$$A_f = \frac{2g_{Af}g_{Vf}}{g_{Af}^2 + g_{Vf}^2}. \quad (38)$$

For $f = b$:

$$A_b = \frac{2g_{Ab}g_{Vb}}{v_b^2 g_{Ab}^2 + (3 - v_b^2)g_{Vb}^2/2}, \quad (39)$$

where v_b is the velocity of the b quark:

$$v_b = \sqrt{1 - \frac{4\hat{m}_b^2}{m_Z^2}}. \quad (40)$$

Here $\hat{m}_b$ is the value of the running mass of the b-quark at scale m_Z calculated in $\overline{MS}$ scheme [36].

The forward–backward charge asymmetry in the decay to $f\bar{f}$ equals:

$$A_{FB}^f = \frac{N_F - N_B}{N_F + N_B} = \frac{3}{4} A_e A_f, \quad (41)$$

where $N_F (N_B)$ is the number of events with f going into the forward (backward) hemisphere; A_e refers to the creation of Z boson in e^+e^- annihilation, while A_f refers to its decay in $f\bar{f}$.

The longitudinal polarization of the τ-lepton in the decay $Z \to \tau\bar{\tau}$ is $P_\tau = -A_\tau$. If, however, the polarization is measured as a function of the angle θ between the momentum of a τ^- and the direction of the electron beam, this allows the determination of not only A_τ, but A_e as well:

$$P_\tau(\cos\theta) = -\frac{A_\tau(1 + \cos^2\theta) + 2A_e\cos\theta}{1 + \cos^2\theta + 2A_\tau A_e}. \quad (42)$$

The polarization P_τ is found from $P_\tau(\cos\theta)$ by separately integrating the numerator and the denominator in equation (42) over the total solid angle.

Table 1.

Observable	Experiment	Born	Pull
m_W (GeV)	80.390(64)	79.956(12)	6.8
m_W/m_Z	0.8816(7)	0.8768(1)	6.8
s_W^2	0.2228(12)	0.2312(2)	−6.8
Γ_l (MeV)	83.90(10)	83.57(1)	3.3
g_{Al}	−0.5010(3)	−0.5000(0)	−3.3
g_{Vl}/g_{Al}	0.0749(9)	0.0754(9)	−0.5
s_l^2	0.2313(2)	0.2312(2)	0.5

The relative difference between total cross section at the Z-peak for the left- and right-polarized electrons that collide with non-polarized positrons (measured at the SLC collider) is

$$A_{LR} \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = A_e. \tag{43}$$

The measurement of parity violating effects allows one to determine experimentally the ratios g_{Vf}/g_{Af}, while the measurements of leptonic and hadronic widths allow to find g_{Af} and $\hat{\alpha}_s$.

Table 1 compares the experimental and the Born values of the so-called 'hadronless' observables m_W, g_{Al} and g_{Vl}. For the reader's convenience the table lists different representations of the same observable known in the literature:

$$s_W^2 = 1 - \frac{m_W^2}{m_Z^2}, \tag{44}$$

$$s_l^2 \equiv s_{eff}^2 \equiv \sin^2\theta_{eff}^{lept} \equiv \frac{1}{4}\left(1 - \frac{g_{Vl}}{g_{Al}}\right). \tag{45}$$

The experimental values in the table are taken from [22], assuming that lepton universality holds. The pull shown in the last column is obtained by dividing the difference 'Exp − Born' by experimental uncertainty (shown in brackets). One can see that the discrepancy between experimental data and Born values are very large for m_W and substantial for g_{Al}. That means that electroweak radiative corrections are essential. As for g_{Vl}/g_{Al}, its experimental and Born values coincide. Moreover the theoretical uncertainty is the same as the experimental one; thus the pull is practically vanishing. Such high experimental accuracy for g_{Vl}/g_{Al} has been achieved only recently. As for m_W and Γ_l, their experimental uncertainties are much larger than the theoretical ones.

We would like to mention that in 1991, when we published our first paper on electroweak corrections to Z-decays, the LEP experimental data were in perfect agreement with the Born predictions of table 1. This demonstrates the remarkable progress in experimental accuracy.

4. One-loop corrections to hadronless observables

4.1. Four types of Feynman diagrams

Four types of Feynman diagrams contribute to electroweak corrections for the observables of interest to us here, m_W/m_Z, g_{Al}, g_{Vl}/g_{Al}:

(1) Self-energy loops for W and Z bosons with virtual ν, l, q, H, W and Z in loops (see figures 3(*b*)–(*n*)).

(2) Loops of charged particles that result in transition of a Z boson into a virtual photon (see figures 3(*o*)–(*r*)).

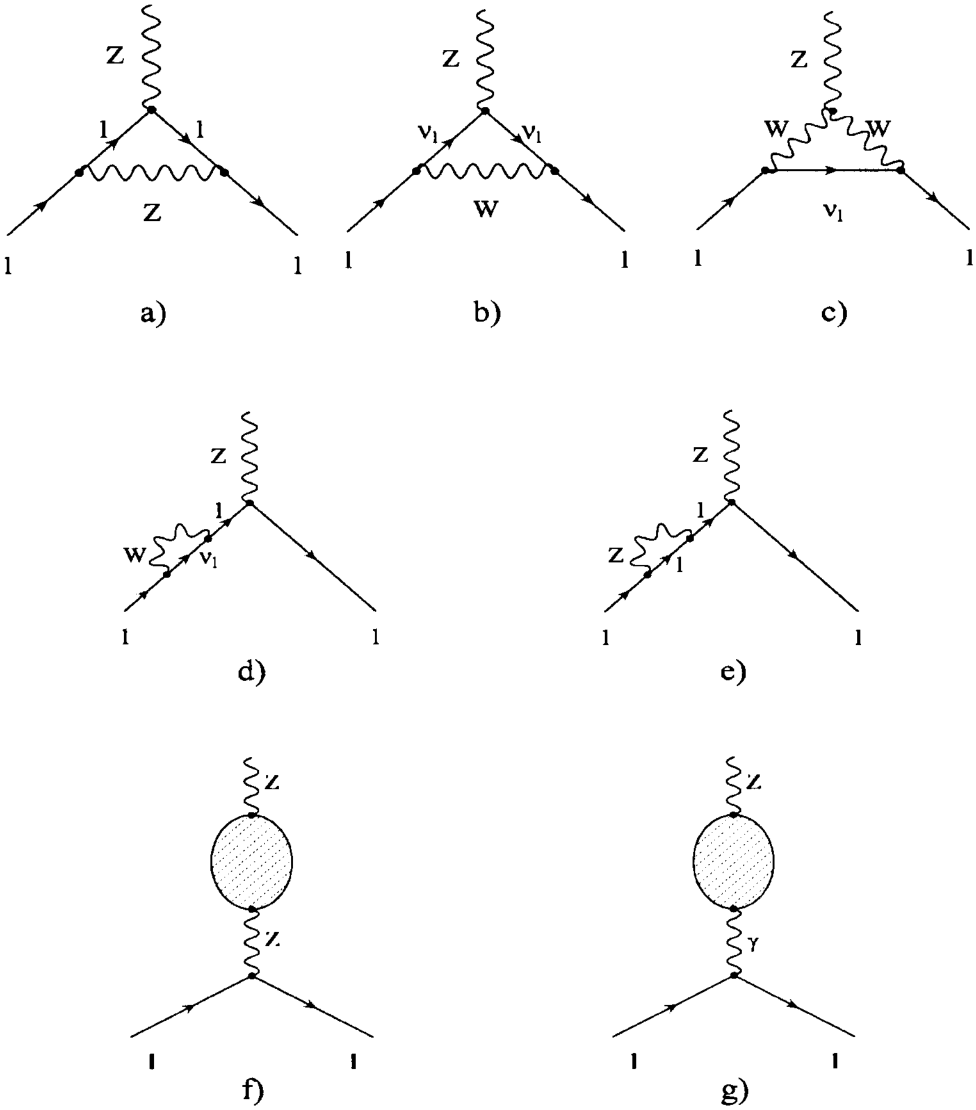

Figure 4. Vertex triangular diagrams in the $Z \to l\bar{l}$ decay (*a*)–(*c*). Loops that renormalize the lepton wavefunctions in the $Z \to l\bar{l}$ decay (of course, the antilepton has similar loops) (*d*), (*e*). Types of diagrams that renormalize the Z boson wavefunction in the $Z \to l\bar{l}$ decay (*f*), (*g*). The virtual particles in the loops are discussed in the text.

(3) Vertex triangles with virtual leptons and a virtual W or Z boson (see figures 4(*a*)–(*c*)).
(4) Electroweak corrections to lepton and Z boson wavefunctions (see figures 4(*d*)–(*g*)).

It must be emphasized that Z boson self-energy loops contribute not only to the mass m_Z and, consequently, to the m_W/m_Z ratio but also to the Z boson decay to $l\bar{l}$, to which $Z \leftrightarrow \gamma$ transitions also contribute because these diagrams give corrections to the Z boson wavefunction. Moreover, there is no simple one-to-one correspondence between Feynman diagrams and amplitudes. This is caused by the choice of G_μ as an input observable which enters the expression for s and c. As a result, e.g., there is a contribution to m_W/m_Z coming from the box and vertex diagrams in the one-loop amplitude of the muon decay. In a similar way the self-energy of the W boson enters the amplitudes for decay $Z \to l\bar{l}$: see also appendix D.

Obviously, electroweak corrections to m_W/m_Z, g_{Al} and g_{Vl}/g_{Al} are dimensionless and thus can be expressed in terms of $\bar{\alpha}$, c, s and the dimensionless parameters

$$t = \left(\frac{m_t}{m_Z}\right)^2, \qquad h = \left(\frac{m_H}{m_Z}\right)^2, \tag{46}$$

where m_t is the mass of the t quark and m_H is the higgs mass. (Masses of leptons and all quarks except t give only very small corrections.)

4.2. *The asymptotic limit at $m_t^2 \gg m_Z^2$*

Following papers by Veltman [37], it became clear that in the limit $t \gg 1$ electroweak radiative corrections are dominated by terms proportional to t. These terms stem from the violation of weak isotopic invariance by the large difference of m_t and m_b (see figures 3(*c*), (*i*) and (*j*)).

After the discovery of the top quark it turned out that experimentally $t \simeq 3.7$. As we shall demonstrate in this review, for such a value of t the contributions of the terms which are not enhanced by the factor t are comparable to the enhanced ones. Still, it is convenient to split the calculation of corrections into a number of stages and begin by calculating the asymptotic limit for $t \gg 1$.

The main contribution comes from diagrams that contain t and b quarks because the large difference of m_t and m_b strongly breaks isotopic invariance. A simple calculation (see appendix D) gives the following result for the sum of the Born and one-loop terms:

$$m_W/m_Z = c + \frac{3c}{32\pi s^2(c^2 - s^2)}\bar{\alpha}t, \tag{47}$$

$$g_{Al} = -\frac{1}{2} - \frac{3}{64\pi s^2 c^2}\bar{\alpha}t, \tag{48}$$

$$R \equiv g_{Vl}/g_{Al} = 1 - 4s^2 + \frac{3}{4\pi(c^2 - s^2)}\bar{\alpha}t, \tag{49}$$

$$g_\nu = \frac{1}{2} + \frac{3}{64\pi s^2 c^2}\bar{\alpha}t. \tag{50}$$

The presence of t-enhanced terms in radiative corrections to Z boson decays allowed the prediction of the mass of the top quark before its actual discovery [38, 39].

4.3. *The functions $V_m(t, h)$, $V_A(t, h)$ and $V_R(t, h)$*

If we now switch from the asymptotic case of $t \gg 1$ to the realistic value of t, then one should make the substitution in equations (47)–(50):

$$t \to t + T_i(t), \tag{51}$$

in which the index $i = m, A, R, \nu$ denotes m_W/m_Z, g_{Al}, $R \equiv g_{Vl}/g_{Al}$ and g_ν, respectively.

The functions T_i are relatively simple combinations of algebraic and logarithmic functions. Their numerical values for a range of values of m_t are given in table 2. The functions $T_i(t)$ thus describe the contribution of the quark doublet t, b to m_W/m_Z, g_A, $R = g_{Vl}/g_{Al}$ and g_ν. If, however, we now take into account the contributions of the remaining virtual particles, then the result can be given in the form

$$t \to V_i(t, h) = t + T_i(t) + H_i(h) + C_i + \delta V_i(t). \tag{52}$$

Here $H_i(h)$ contain the contribution of the virtual vector and higgs bosons W, Z and H and are functions of the higgs mass m_H. (The mass of the W boson enters $H_i(h)$ via the parameter

Table 2.

m_t (GeV)	t	T_m	T_A	T_R
120	1.732	0.323	0.465	0.111
130	2.032	0.418	0.470	0.154
140	2.357	0.503	0.473	0.193
150	2.706	0.579	0.476	0.228
160	3.079	0.649	0.478	0.261
170	3.476	0.713	0.480	0.291
180	3.896	0.772	0.481	0.319
190	4.341	0.828	0.483	0.345
200	4.810	0.880	0.484	0.370
210	5.303	0.929	0.485	0.393
220	5.821	0.975	0.485	0.415
230	6.362	1.019	0.486	0.436
240	6.927	1.061	0.487	0.456

Table 3.

m_H (GeV)	h	H_m	H_A	H_R
0.01	0.000	1.120	−8.716	1.359
0.10	0.000	1.119	−5.654	1.354
1.00	0.000	1.103	−2.652	1.315
10.00	0.012	0.980	−0.133	1.016
50.00	0.301	0.661	0.645	0.360
100.00	1.203	0.433	0.653	−0.022
150.00	2.706	0.275	0.588	−0.258
200.00	4.810	0.151	0.518	−0.430
250.00	7.516	0.050	0.452	−0.566
300.00	10.823	−0.037	0.392	−0.679
350.00	14.732	−0.112	0.338	−0.776
400.00	19.241	−0.178	0.289	−0.860
450.00	24.352	−0.238	0.244	−0.936
500.00	30.065	−0.292	0.202	−1.004
550.00	36.378	−0.341	0.164	−1.065
600.00	43.293	−0.387	0.128	−1.122
650.00	50.809	−0.429	0.095	−1.175
700.00	58.927	−0.469	0.064	−1.223
750.00	67.646	−0.506	0.035	−1.269
800.00	76.966	−0.540	0.007	−1.311
850.00	86.887	−0.573	−0.019	−1.352
900.00	97.410	−0.604	−0.044	−1.390
950.00	108.534	−0.633	−0.067	−1.426
1000.00	120.259	−0.661	−0.090	−1.460

c, defined by equation (25)). The explicit form of the functions H_i is given in [41, 42] and their numerical values for various values of m_H are given in table 3.

The constants C_i in equation (52) include the contributions of light fermions to the self-energy of the W and Z bosons, and also to the Feynman diagrams, describing the electroweak corrections to the muon decay, as well as triangle diagrams, describing the Z boson decay. The constants C_i are relatively complicated functions of s^2 (see [41, 42]). We list here their

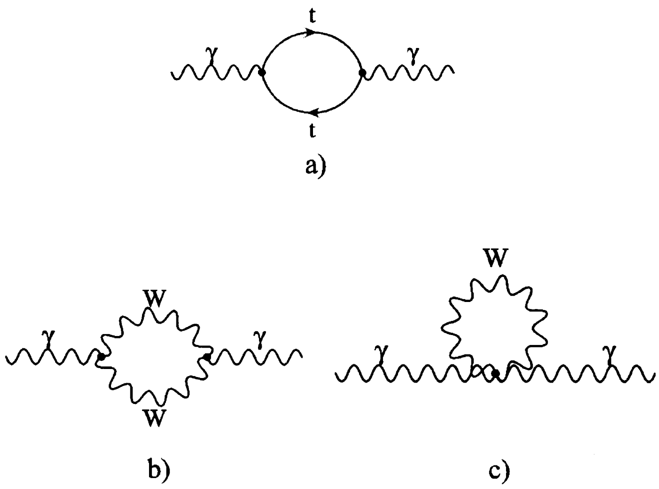

Figure 5. Virtual t quarks (a) and W bosons (b), (c) in the photon polarization of the vacuum.

numerical values for $s^2 = 0.231\,10 - \delta s^2$:

$$C_m = -1.3497 + 4.13\delta s^2, \tag{53}$$

$$C_A = -2.2621 - 2.63\delta s^2, \tag{54}$$

$$C_R = -3.5045 - 5.72\delta s^2, \tag{55}$$

$$C_\nu = -1.1641 - 4.88\delta s^2. \tag{56}$$

4.4. Corrections $\delta V_i(t)$

Finally, the last term in equation (52) includes the sum of corrections of three different types. Their common feature is that they do not contain more than one electroweak loop:

$$\delta V_i = \delta_1 V_i + \delta_2 V_i + \delta_3 V_i. \tag{57}$$

(1) The corrections $\delta_1 V_i$ are extremely small. They contain contributions of the W boson and the t quark to the polarization of the electromagnetic vacuum $\delta_W\alpha$ and $\delta_t\alpha$, which traditionally are not included into the running of $\alpha(q^2)$, i.e. into $\bar{\alpha}$ (see figure 5). It is reasonable to treat them as electroweak corrections. This is especially true for the W-loop that depends on the gauge chosen for the description of the W and Z bosons. Only after this loop is taken into account do the resultant electroweak corrections become gauge-invariant, as it should indeed be for physical observables. Here and hereafter in the calculations the 't Hooft–Feynman gauge is used:

$$\delta_1 V_m(t,h) = -\frac{16}{3}\pi s^4 \frac{1}{\alpha}(\delta_W\alpha + \delta_t\alpha) = -0.055, \tag{58}$$

$$\delta_1 V_R(t,h) = -\frac{16}{3}\pi s^2 c^2 \frac{1}{\alpha}(\delta_W\alpha + \delta_t\alpha) = -0.181, \tag{59}$$

$$\delta_1 V_A(t,h) = \delta_1 V_\nu(t,h) = 0, \tag{60}$$

where

$$\delta_W\alpha = 0.000\,50, \tag{61}$$

$$\delta_t\alpha \simeq -0.000\,05(1). \tag{62}$$

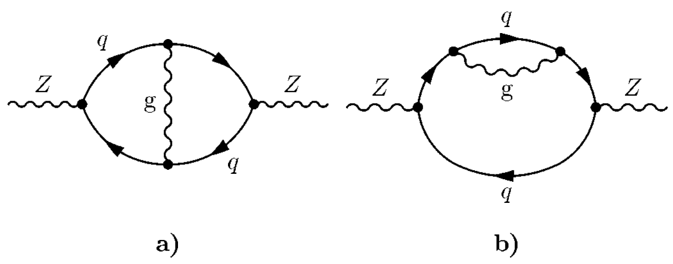

Figure 6. Gluon corrections to the electroweak quark loop of the Z boson self-energy.

(See equations (B.18) and (B.17) from appendix B. Unless specified otherwise, we use $m_t = 175$ GeV in numerical evaluations.)

(2) The corrections $\delta_2 V_i$ are the largest ones. They are caused in the order $\bar{\alpha}\hat{\alpha}_s$ by virtual gluons in electroweak loops of light quarks $q = u, d, s, c, b$ and heavy quark t (see figure 6):

$$\delta_2 V_i(t) = \delta_2^q V_i + \delta_2^t V_i(t). \tag{63}$$

Due to asymptotic freedom of QCD [40] these corrections were calculated in perturbation theory. The analytical expressions for corrections $\delta_2^q V_i$ and $\delta_2^t V_i(t)$ are given in [41, 42]. Here we only give numerical estimates for them,

$$\delta_2^q V_m = -0.377 \frac{\hat{\alpha}_s}{\pi}, \tag{64}$$

$$\delta_2^q V_A = 1.750 \frac{\hat{\alpha}_s}{\pi}, \tag{65}$$

$$\delta_2^q V_R = 0, \tag{66}$$

$$\delta_2^t V_m(t) = -11.67 \frac{\hat{\alpha}_s(m_t)}{\pi} = -10.61 \frac{\hat{\alpha}_s}{\pi}, \tag{67}$$

$$\delta_2^t V_A(t) = -10.10 \frac{\hat{\alpha}_s(m_t)}{\pi} = -9.18 \frac{\hat{\alpha}_s}{\pi}, \tag{68}$$

$$\delta_2^t V_R(t) = -11.88 \frac{\hat{\alpha}_s(m_t)}{\pi} = -10.80 \frac{\hat{\alpha}_s}{\pi}, \tag{69}$$

where [40]

$$\hat{\alpha}_s(m_t) = \frac{\hat{\alpha}_s}{1 + \frac{23}{12\pi}\hat{\alpha}_s \log t}. \tag{70}$$

(For numerical evaluation, we use $\hat{\alpha}_s \equiv \hat{\alpha}_s(m_Z) = 0.120$.) We have mentioned already that the corrections $\delta_2^t V_i(t)$, whose numerical values were given in (67)–(69), are much larger than all other terms included in δV_i. We emphasize that the term in $\delta_2^t V_i$ that is leading for high t is universal: it is independent of i. As shown in [43], this leading term is obtained by multiplying the Veltman asymptotics t by a factor

$$1 - \frac{2\pi^2 + 6}{9} \frac{\hat{\alpha}_s(m_t)}{\pi}, \tag{71}$$

or, numerically,

$$t \to t \left(1 - 2.86 \frac{\hat{\alpha}_s(m_t)}{\pi}\right). \tag{72}$$

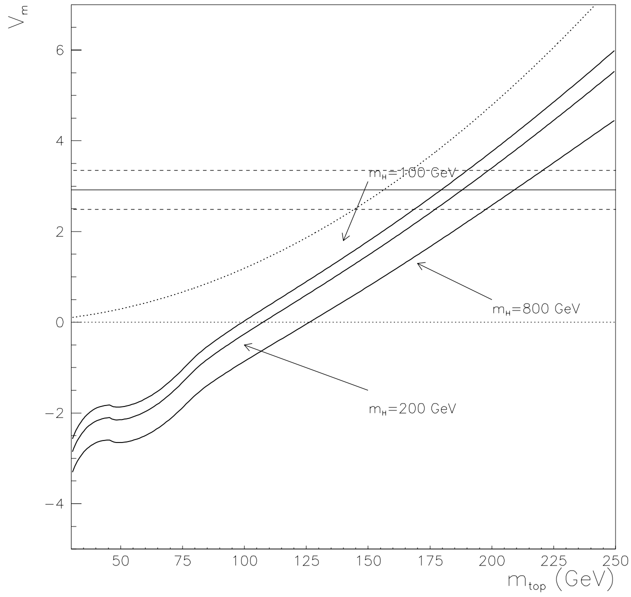

Figure 7. V_m as a function of m_t for three values of m_H. The dotted parabola corresponds to Veltman approximation: $V_m = t$. The solid horizontal line traces the experimental value of V_m while the dashed horizontal lines give its upper and lower limits at the 1σ level.

Qualitatively, the factor (71) corresponds to the fact that the running mass of the t quark at momenta $p^2 \sim m_t^2$ that circulate in the t quark loop is lower than the 'on the mass-shell' mass of the t quark. It is interesting to compare the correction (72) with the quantity

$$\tilde{m}_t^2 \equiv m_t^2(p_t^2 = -m_t^2) = m_t^2 \left(1 - 2.78 \frac{\hat{\alpha}_s(m_t)}{\pi}\right), \tag{73}$$

calculated in the Landau gauge in [44], p 102. The agreement is overwhelming. There is, therefore, a simple mnemonic rule for evaluating the main gluon corrections for the t-loop.

(3) Corrections $\delta_3 V_i$ of the order of $\bar{\alpha}\hat{\alpha}_s^2$ are extremely small. They were calculated in the literature [45] for the term leading in t (i.e. $\bar{\alpha}\hat{\alpha}_s^2 t$). They are independent of i (in numerical estimates we use for the number of light quark flavours $N_f = 5$):

$$\delta_3 V_i(t) \simeq -(2.38 - 0.18 N_f)\hat{\alpha}_s^2(m_t)t \simeq -1.48\hat{\alpha}_s^2(m_t)t = -0.07. \tag{74}$$

4.5. Accidental (?) compensation and the mass of the t quark

Now that we have expressions for all terms in equation (52), it will be convenient to analyse their roles and the general behaviour of the functions $V_i(t, h)$. As functions of m_t at three fixed values of m_H, they are shown in figures 7–9. In all these figures, we see a cusp at $m_t = m_Z/2$. This is a typical threshold singularity that arises when the channel $Z \to t\bar{t}$ is opened. It is of no practical significance since experiments give $m_t \simeq 175$ GeV. What really impresses is that the function V_R vanishes at this value of m_t. This happens because of the compensation of the leading term t and the rest of the terms which produce a negative aggregate contribution, the main negative contribution coming from the light fermions (see equation (55) for the constant C_R).

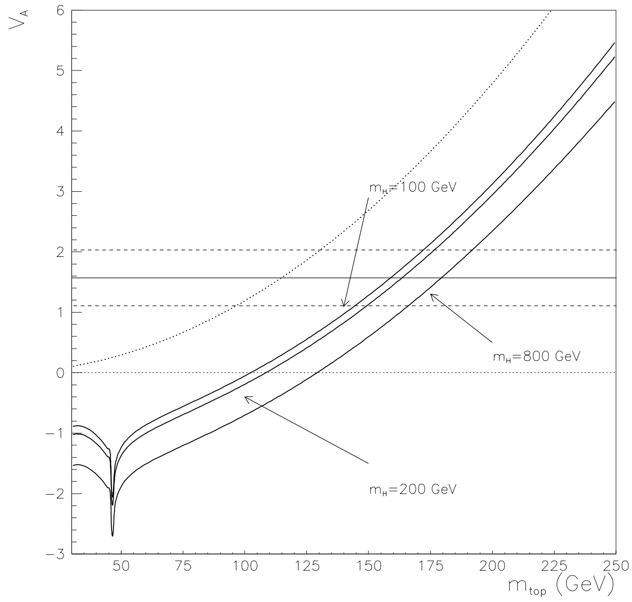

Figure 8. V_A as a function of m_t. The dotted parabola corresponds to Veltman approximation: $V_A = t$. The solid horizontal line traces the experimental value of V_A while the dashed horizontal lines give its upper and lower limits at the 1σ level.

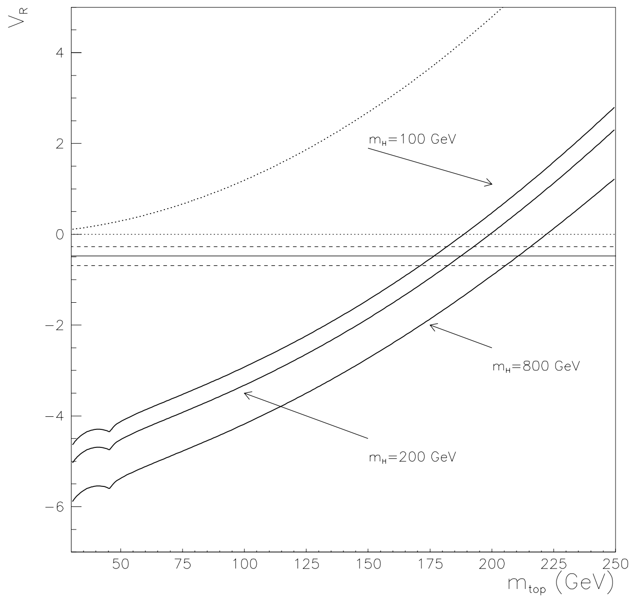

Figure 9. V_R as a function of m_t. The dotted parabola corresponds to Veltman approximation: $V_R = t$. The solid horizontal line traces the experimental value of V_R while the dashed horizontal lines give its upper and lower limits at the 1σ level.

In the one-electroweak loop approximation each function $V_i(t, h)$ is a sum of two functions, one of which is t-dependent but independent of h, while the other is h-dependent but independent of t (plus, of course, a constant which is independent of both t and h). Therefore, the curves for $m_H = 100$ and 800 GeV in figures 7–9 are produced by the parallel transfer of the curve for $m_H = 200$ GeV.

We see in figure 9 that if the t quark were light, radiative corrections would be large and negative, and if it were very heavy, they would be large and positive. This looks like a conspiracy of the observable mass of the t quark and all other parameters of the electroweak theory, as a result of which the electroweak correction V_R becomes anomalously small.

One should specially note the dashed parabola in figures 7–9 corresponding to the Veltman term t. We see that in the interval $0 < m_t < 250$ GeV it lies much higher than V_A and V_R and approaches V_m only in the right-hand side of figure 7. Therefore, the so-called non-leading 'small' corrections that were typically replaced with ellipses in standard texts, are found to be comparable with the leading term t.

A glance at figure 9 readily explains how the experimental analysis of electroweak corrections allowed, despite their smallness, a prediction, within the framework of the MSM, of the t quark mass. Even when the experimental accuracy of LEP-I and SLC experiments was not sufficient for detecting electroweak corrections, it was sufficient for establishing the t quark mass using the points at which the curves $V_R(m_t)$ intersect the horizontal line corresponding to the experimental value of V_R and the thin lines parallel to it that show the band of one standard deviation. The accuracy in determining m_t is imposed by the band width and the slope of $V_R(m_t)$.

The dependence $V_i(m_H)$ for three fixed values of $m_t = 150$, 175 and 200 GeV can be presented in a similar manner. As follows from the explicit form of the terms $H_i(m_H)$, the dependence $V_i(m_H)$ is considerably less steep (it is logarithmic). This is the reason why the prediction of the higgs mass extracted from electroweak corrections has such a high uncertainty. The accuracy of prediction of m_H greatly depends on the value of the t quark mass. If $m_t = 150 \pm 5$ GeV, then $m_H < 80$ GeV at the 3σ level. If $m_t = 200 \pm 5$ GeV, then $m_H > 150$ GeV at the 3σ level. If, however, $m_t = 175 \pm 5$ GeV, as given by FNAL experiments [27], we are hugely unlucky: the constraint on m_H is rather mild (see figure 10).

Before starting a discussion of hadronic decays of the Z boson, let us 'go back to the roots' and recall how the equations for $V_i(m_t, m_H)$ were derived.

4.6. How to calculate V_i? 'Five steps'

An attentive reader should have already come up with the question: what makes the amplitudes of the lepton decays of the Z boson in the one-loop approximation depend on the self-energy of the W boson? Indeed, the loops describing the self-energy of the W boson appear in the decay diagrams of the Z boson only beginning with the two-loop approximation. The answer to this question was already given at the beginning of section 4. We have already emphasized that we find expressions for radiative corrections to Z boson decays in terms of $\bar{\alpha}$, m_Z and G_μ. However, the expression for G_μ includes the self-energy of the W boson even in the one-loop approximation. The point is that we express some observables (in this particular case, m_W/m_Z, g_{Al}, g_{Vl}/g_{Al}) in terms of other, more accurately measured, observables ($\bar{\alpha}$, m_Z, G_μ).

Let us trace how this is achieved, step by step. There are altogether 'five steps to happiness', based on the one-loop approximation. All necessary formulae can be found in appendix D.

Step I. We begin with the electroweak Lagrangian after it has undergone the spontaneous violation of the $SU(2) \times U(1)$ symmetry by the higgs vacuum condensate VEV η and the W

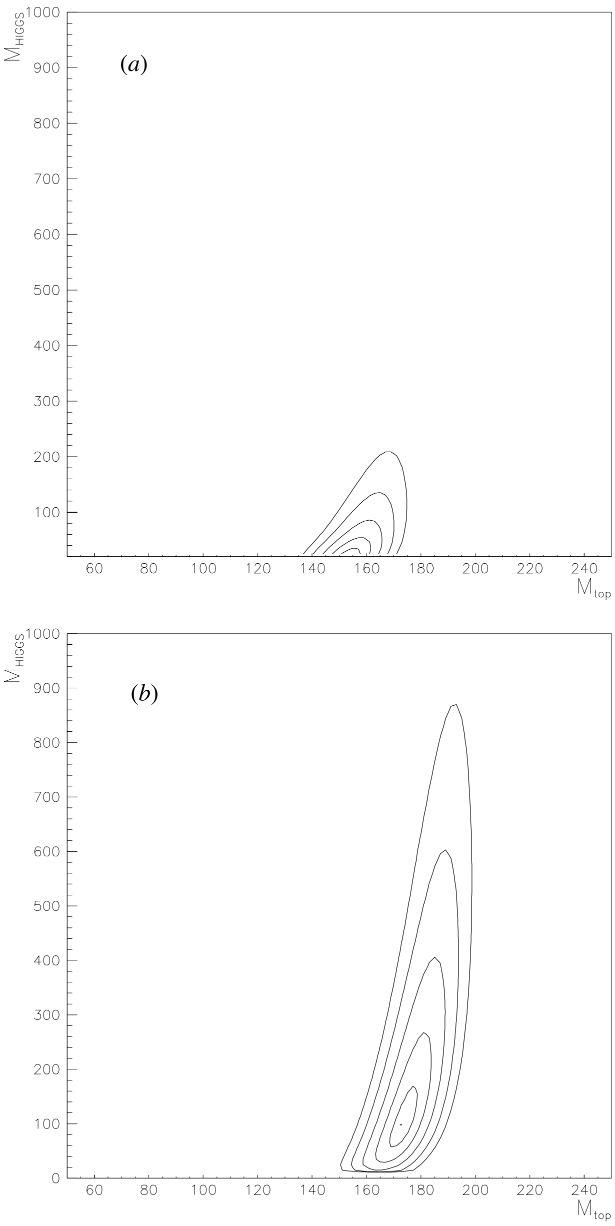

Figure 10. $m_t - m_H$ exclusion plots with assumptions of: (*a*) $m_t = 150(5)$ GeV; (*b*) $m_t = 175(5)$ GeV; (*c*) $m_t = 200 \pm 5$ GeV.

and Z bosons became massive. Let us consider the bare coupling constants (the bare charges e_0 of the photon, g_0 of the W boson and f_0 of the Z boson) and the bare masses of the vector bosons:

$$m_{Z0} = \tfrac{1}{2} f_0 \eta, \tag{75}$$

$$m_{W0} = \tfrac{1}{2} g_0 \eta, \tag{76}$$

and also bare masses: m_{t0} of the t quark and m_{H0} of the higgs.

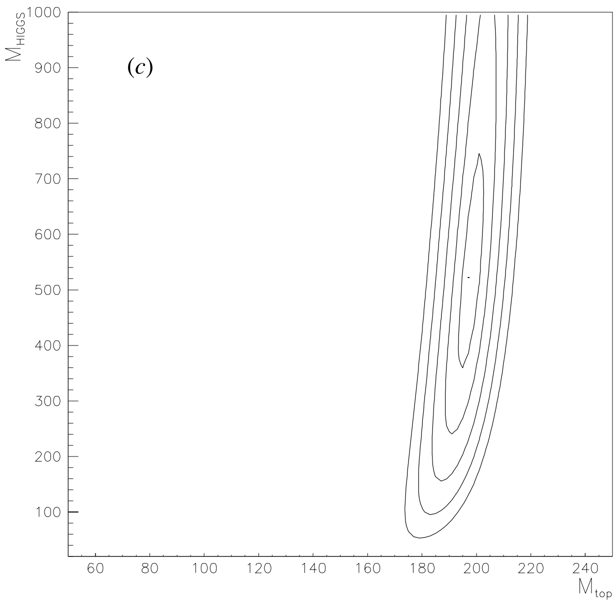

Figure 10. (Continued.)

Step II. We express $\bar{\alpha}$, G_μ, m_Z in terms of $f_0, g_0, e_0, \eta, m_{t0}, m_{H0}$ and $1/\varepsilon$ (see appendix D). Here $1/\varepsilon$ appears because we use the dimensional regularization, calculating Feynman integrals in the space of D dimensions (see appendix A). These integrals diverge at $D = 4$ and are finite in the vicinity of $D = 4$. By definition,

$$2\varepsilon = 4 - D \to 0. \tag{77}$$

Note that in the one-loop approximation $m_{t0} = m_t$, $m_{H0} = m_H$, since we neglect the electroweak corrections to the masses of the t quark and the higgs.

Step II is almost physics: we calculate Feynman diagrams (we say 'almost' to emphasize that observables are expressed in terms of non-observable, 'bare', and generally infinite quantities).

Step III. Let us invert the expressions derived at step II and write f_0, g_0, η in terms of $\bar{\alpha}$, G_μ, m_Z, m_t, m_H and $1/\varepsilon$. This step is a pure algebra.

Step IV. Let us express V_m, V_A, V_R (or the electroweak one-loop correction to any other electroweak observable, all of them being treated on an equal basis) in terms of f_0, g_0, η, m_t, m_H and $1/\varepsilon$. (Like step II, this step is again almost physics.)

Step V. Let us express V_m, V_A, V_R (or any other electroweak correction) in terms of $\bar{\alpha}$, G_μ, m_Z, m_t, m_H using the results of steps III and IV. Formally this is pure algebra, but in fact pure physics, since now we have expressed certain physical observables in terms of other observables. If no errors were made on the way, the terms $1/\varepsilon$ cancel out. As a result, we arrive at formula (52) which gives V_i as elementary functions of t, h and s.

The five steps outlined above are very simple and visually clear. We obtain the main relations without using the 'heavy artillery' of quantum field theory with its counterterms in the Lagrangian and the renormalization procedure. This simplicity and visual clarity became possible owing to the one-loop electroweak approximation. (Even though this approach to renormalization is possible in multiloop calculations, it becomes more cumbersome than the standard procedures.) As for the QCD corrections to quark electroweak loops hidden in the terms δV_i in equation (52), we take the relevant formulae from the calculations of other authors.

5. One-loop corrections to hadronic decays of the Z boson

5.1. The leading quarks and hadrons

As discussed above (see formulae (31)–(37)), an analysis of hardronic decays reduces to the calculation of decays to pairs of quarks: $Z \to q\bar{q}$. The key role is played by the concept of leading hadrons that carry away the predominant part of the energy. For example, the $Z \to c\bar{c}$ decay mostly produces two hadron jets flying in opposite directions, in one of which the leading hadron is the one containing the $\bar{c}$-quark, for example, $D^- = \bar{c}d$, and in the other the hadron with the c-quark, for example, $D^0 = c\bar{u}$ or $\Lambda_c^+ = udc$. Likewise, $Z \to b\bar{b}$ decays are identified by the presence of high-energy B or $\bar{B}$ mesons. If one selects only particles with energy close to $m_Z/2$, the identification of the initial quark channels is unambiguous. The total number of such cases will, however, be small. If one takes into account as a signal less energetic B mesons, one faces the problem of their origin. Indeed, a pair $b\bar{b}$ can be created not only directly by a Z boson but also by a virtual gluon in, say, a $Z \to c\bar{c}$ decay or $Z \to u\bar{u}$, or $s\bar{s}$. This example shows the sort of difficulty encountered by experimentalists trying to identify a specific quark–antiquark channel. Furthermore, owing to such secondary pairs, the total hadron width is not strictly equal to the sum of partial quark widths.

We remind the reader that for the partial width Γ_q of the $Z \to q\bar{q}$ decay we had equation (31), where the standard width Γ_0 was given by equation (29) and the radiators R_{Aq} and R_{Vq} are given in appendix E. As for the electroweak corrections, they are included in the coefficients g_{Aq} and g_{Vq}. The sum of the Born and one-loop terms has the form

$$g_{Aq} = T_{3q}\left[1 + \frac{3\bar{\alpha}}{32\pi s^2 c^2} V_{Aq}(t, h)\right], \tag{78}$$

$$R_q \equiv g_{Vq}/g_{Aq} = 1 - 4|Q_q|s^2 + \frac{3|Q_q|}{4\pi(c^2 - s^2)}\bar{\alpha} V_{Rq}(t, h). \tag{79}$$

5.2. Decays to pairs of light quarks

Here, as in the case of hadronless observables, the quantities V that characterize corrections are normalized in the standard way: $V \to t$ as $t \gg 1$. Naturally, those terms in V that are due to the self-energies of vector bosons are identical for both leptons and quarks. The deviation of the differences $V_{Aq} - V_{Al}$ and $V_{Rq} - V_{Rl}$ from zero are caused by the differences in radiative corrections to vertices $Z \to q\bar{q}$ and $Z \to l\bar{l}$. For four light quarks we have

$$V_{Au}(t, h) = V_{Ac}(t, h) = V_{Al}(t, h) + \left[\frac{128\pi s^3 c^3}{3\bar{\alpha}}(F_{Al} + F_{Au}) = 0.2634\right], \tag{80}$$

$$V_{Ad}(t, h) = V_{As}(t, h) = V_{Al}(t, h) + \left[\frac{128\pi s^3 c^3}{3\bar{\alpha}}(F_{Al} - F_{Ad}) = 0.6295\right], \tag{81}$$

$$\begin{aligned} V_{Ru}(t, h) = V_{Rc}(t, h) = V_{Rl}(t, h) \\ + \left[\frac{16\pi sc(c^2 - s^2)}{3\bar{\alpha}}\left[F_{Vl} - (1 - 4s^2)F_{Al} + \frac{3}{2}\left(-\left(1 - \frac{8}{3}s^2\right)F_{Au} + F_{Vu}\right)\right]\right. \\ \left. = 0.1220\right], \end{aligned} \tag{82}$$

$$V_{Rd}(t, h) = V_{Rs}(t, h) = V_{Rl}(t, h)$$

$$+\left[\frac{16\pi sc(c^2-s^2)}{3\bar{\alpha}}\left[F_{Vl}-(1-4s^2)F_{Al}+3\left(\left(1-\frac{4}{3}s^2\right)F_{Ad}-F_{Vd}\right)\right]\right.$$

$$\left.=0.2679\right], \tag{83}$$

where (see [41, 42]):

$$F_{Al}=\frac{\bar{\alpha}}{4\pi}(3.0099+16.4\delta s^2), \tag{84}$$

$$F_{Vl}=\frac{\bar{\alpha}}{4\pi}(3.1878+14.9\delta s^2), \tag{85}$$

$$F_{Au}=-\frac{\bar{\alpha}}{4\pi}(2.6802+14.7\delta s^2), \tag{86}$$

$$F_{Vu}=-\frac{\bar{\alpha}}{4\pi}(2.7329+14.2\delta s^2), \tag{87}$$

$$F_{Ad}=\frac{\bar{\alpha}}{4\pi}(2.2221+13.5\delta s^2), \tag{88}$$

$$F_{Vd}=\frac{\bar{\alpha}}{4\pi}(2.2287+13.5\delta s^2). \tag{89}$$

The values of F are given here for $s^2 = 0.231\,10 - \delta s^2$. The accuracy to five decimal places is purely arithmetic. The physical uncertainties introduced by neglecting higher-order loops manifest themselves already in the third decimal place.

In addition to the changes given by equations (80)–(83), one has to also take into account emission of a virtual or ‘free’ gluon from a vertex quark triangle.

The corresponding effect cannot be parametrized in terms V_{Aq} and V_{Rq}, because it contributes also to the radiators R_{Aq} and R_{Vq}. The change of Γ_h caused by it has been calculated only recently [46] and turned out to be rather small:

$$\delta\Gamma_h(Z\to u,d,s,c)=-0.59(3)\ \text{MeV}. \tag{90}$$

5.3. *Decays to $b\bar{b}$ pair*

In the $Z \to b\bar{b}$ decay it is necessary to take into account additional t-dependent vertex corrections:

$$V_{Ab}(t,h)=V_{Ad}(t,h)-\left[\frac{8s^2c^2}{3(3-2s^2)}(\phi(t)+\delta_{\alpha_s}\phi(t))=5.03\right], \tag{91}$$

$$V_{Rb}(t,h)=V_{Rd}(t,h)-\left[\frac{4s^2(c^2-s^2)}{3(3-2s^2)}(\phi(t)+\delta_{\alpha_s}\phi(t))=1.76\right]. \tag{92}$$

Here the term $\phi(t)$ calculated in [47] corresponds to a $t\bar{t}W$ vertex triangle (see figure 11(a)), while the term $\delta_{\alpha_s}\phi(t)$ calculated in [48], corresponds to the leading gluon corrections to the term $\phi(t)$ (see figure 11(b)): $\delta_{\alpha_s}\phi(t) \sim \alpha_s t$. Expressions for $\phi(t)$ and $\delta_{\alpha_s}\phi(t)$ are given in [41, 42]. For $m_t = 175$ GeV, $\hat{\alpha}_s(m_Z) = 0.120$

$$\phi(t)=29.96, \tag{93}$$

$$\delta_{\alpha_s}\phi(t)=-3.02, \tag{94}$$

and correction terms in equations (91) and (92) are very large. The subleading gluon corrections to $\phi(t)$ calculated recently [49] are very small: $\delta\Gamma_h(Z \to b) = -0.04$ MeV.

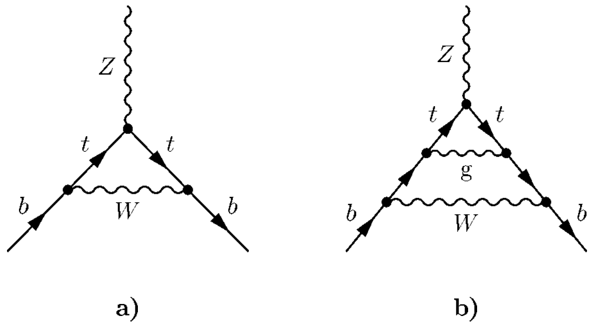

Figure 11. The vertex electroweak diagrams involving t quark and contributing to the $Z \to b\bar{b}$ decay. (b) represents gluon corrections to diagram (a).

6. Comparison of one-electroweak-loop results and experimental LEP-I and SLC data

6.1. LEPTOP *code*

A number of computer programs (codes) were written for comparing high-precision data of LEP-I and SLC. The best known of these programs in Europe is ZFITTER [50], which takes into account not only electroweak radiative corrections but also all purely electromagnetic ones, including, among others, the emission of photons by colliding electrons and positrons. Some of the first publications in which the t quark mass was predicted on the basis of precision measurements [51], were based on the code ZFITTER. Other European codes, BHM, WOH [52], TOPAZO [53], somewhat differ from ZFITTER. The best known in the USA are the results generated by the code used by Erler and Langacker [23, 54].

The original idea of the authors of this review in 1991–3 was to derive simple analytical formulae for electroweak radiative corrections, which would make it possible to predict the t quark mass using no computer codes, just by analysing experimental data on a sheet of paper. Alas, the diversity of hadron decays of Z bosons, depending on the constants of strong gluon interaction $\hat{\alpha}_s$, was such that it was necessary to convert analytical formulae into a computer program which we jokingly dubbed LEPTOP [55]. The LEPTOP calculates the electroweak observables in the framework of the MSM and fits experimental data so as to determine the quantities m_t, m_H and $\hat{\alpha}_s(m_Z)$. The logical structure of LEPTOP is clear from the preceding sections of this review and is shown in the flowchart on page 42. The code of LEPTOP can be downloaded from the Internet home page: http://cppm.in2p3.fr./leptop/intro_leptop.html

A comparison of the codes ZFITTER, BHM, WOH, TOPAZO and LEPTOP carried out in 1994–5 [20] has demonstrated that their predictions for all electroweak observables coincide with accuracy that is much better than the accuracy of the experiment. The flowcharts of LEPTOP and ZFITTER are compared on pages 25 and 27 of [20]; numerical comparison of five codes (their 1995 versions) for twelve observables is presented in figures 11–23 of the same reference. The results of processing the experimental data using LEPTOP are shown below.

6.2. One-loop general fit

The second column of table 4 shows experimental values of the electroweak observables, obtained by averaging the results of four LEP detectors (a), and also SLC data (b) and the data on W boson mass (c). (The data on the W boson mass from the $p\bar{p}$-colliders and LEP-II are also shown, for the reader's convenience, in the form of s_W^2, while the data on s_W^2 from νN-experiments are also shown in the form of m_W. These two numbers are given in

Table 4. Fit of the experimental data [22] with one-electroweak-loop formulae. $m_Z = 91.1867(21)$ GeV is used as an input. Output of the fit: $m_H = 139.1^{+134.2}_{-76.5}$ GeV, $\hat{\alpha}_s = 0.1195 \pm 0.0030$, $\chi^2/n_{d.o.f.} = 15.1/14$.

Observable	Experimental data	Standard model	Pull
(a) LEP			
Shape of Z-peak and lepton asymmetries:			
Γ_Z (GeV)	2.4939(24)	2.4959(18)	−0.8
σ_h (nb)	41.491(58)	41.472(16)	0.3
R_l	20.765(26)	20.747(20)	0.7
A^l_{FB}	0.0168(10)	0.0161(3)	0.8
τ-polarization:			
A_τ	0.1431(45)	0.1465(14)	−0.8
A_e	0.1479(51)	0.1465(14)	0.3
Results for b and c quarks:			
R_b[a]	0.2166(7)	0.2158(2)	1.0
R_c[a]	0.1735(44)	0.1723(1)	0.3
A^b_{FB}	0.0990(21)	0.1027(10)	−1.8
A^c_{FB}	0.0709(44)	0.0734(8)	−0.6
Charge asymmetry for pairs of light quarks $q\bar{q}$:			
$s^2_l(Q_{FB})$	0.2321(10)	0.2316(2)	0.5
(b) SLC			
A_{LR}	0.1504(23)	0.1465(14)	1.7
$s^2_l(A_{LR})$	*0.2311(3)*	*0.2316(2)*	*−1.7*
R_b[a]	0.2166(7)	0.2158(2)	0.9
R_c[a]	0.1735(44)	0.1723(1)	0.3
A_b	0.8670(350)	0.9348(1)	−1.9
A_c	0.6470(400)	0.6676(6)	−0.5
(c) $p\bar{p}$+ LEP-II +νN			
m_W (GeV) ($p\bar{p}$) + LEP-II	80.39(6)	80.36(3)	0.5
	0.2228(13)		
s^2_W (νN)	0.2254(21)	0.2234(6)	0.9
	80.2547(1089)		
m_t (GeV)	173.8(5.0)	171.6(4.9)	0.4

[a] Experimental values of R_b and R_c correspond to the average of LEP-I and SLC results.

italics, emphasizing that they are not independent experimental data. The same refers to s^2_l (A_{LR}).) We take experimental data from the paper [22]. The experimental data of table 4 are used for determining (fitting) the parameters of the standard model in one-electroweak-loop approximation: $m_t, m_H, \hat{\alpha}_s(m_Z)$ and $\bar{\alpha}$. (In fitting m_t, the direct measurements of m_t by CDF and D0 (Collaborations) [27] are also used. In fitting $\bar{\alpha}$, its value from equation (14) was used.) The third column shows the results of the fit of electroweak observables with one loop electroweak formulae. The last column shows the value of the 'pull'. By definition, the pull is the difference between the experimental and the theoretical values divided by experimental uncertainty. The pull values show that for most observables the discrepancy is less than 1σ. The number of degrees of freedom is $18 - 4 = 14$.

Table 5. Recalculation with LEPTOP of table 30 of the EWWG report [22].

Observable	s_l^2	Average over groups of observations	Cumulative average	$\chi^2/n_{d.o.f.}$
A_{FB}^l	0.231 17(55)			
A_τ	0.232 02(57)			
A_e	0.231 41(65)	0.231 53(34)	0.231 53(34)	1.2/2
A_{FB}^b	0.232 26(38)			
A_{FB}^c	0.232 23(112)	0.232 26(36)	0.231 87(25)	3.4/4
$\langle Q_{FB}\rangle$	0.232 10(100)	0.232 10(100)	0.231 89(24)	3.4/5
A_{LR} (SLD)	0.231 09(30)	0.231 09(30)	0.231 57(19)	7.8/6

Table 5† gives experimental values of s_l^2. The third column was obtained by averaging of the second column, and the fourth by cumulative averaging of the third; it also lists the values of χ^2 over the number of degrees of freedom.

7. Two-loop electroweak corrections and theoretical uncertainties

In this section we will discuss heavy top corrections of the second order in α_W to m_W and to coupling constants of Z boson with fermions. Full calculation of α_W^2 corrections is still absent. What have been calculated are corrections of the order $\alpha_W^2\ t^2 = \alpha_W^2(m_t/m_Z)^4$ [56, 57] and corrections $\sim\alpha_W^2 t$ [58–60].

There are two sources of $\alpha_W^2 t^2$ corrections in our approach. The first source are reducible diagrams with top quark in each loop. The second source are irreducible two-loop Feynman diagrams which contain top quark [56, 57]. We start our consideration with the first source the contribution of which is proportional to $(\Pi_Z(0) - \Pi_W(0))^2$. Detailed calculations are presented in appendix F.

7.1. $\alpha_W^2 t^2$ corrections to m_W/m_Z, g_A and g_V/g_A from reducible diagrams

We start our consideration from the ratio of vector boson masses. From equations (F.12) and (F.13) we obtain:

$$\frac{m_W}{m_Z} = c\left[1 + \frac{c^2}{2(c^2 - s^2)}\delta + \frac{3c^4 - 10c^4 s^2}{8(c^2 - s^2)^3}\delta^2\right]. \qquad (95)$$

Substituting the expression for δ from (F.10) and using the definition of V_m from equations (47), (52) we obtain the following correction to the function V_m:

$$\delta_4' V_m = \frac{4\pi s^2 c^4 (3 - 10s^2)}{3\bar{\alpha}(c^2 - s^2)^2}\delta^2 = \frac{3(3 - 10s^2)\bar{\alpha} t^2}{64\pi s^2 (c^2 - s^2)^2}. \qquad (96)$$

The correction to axial coupling constant g_{Al} is easily derived from equations (F.14) (since $g_{Al} \sim f_0$), (F.10) and the definition of V_{Al}, equations (48), (52):

$$g_{Al} = -\tfrac{1}{2} - \tfrac{1}{4}\delta - \tfrac{3}{16}\delta^2, \qquad (97)$$

$$\delta_4' V_A = \frac{9\bar{\alpha} t^2}{64\pi s^2 c^2}. \qquad (98)$$

† Table 5 is our recalculation with a LEPTOP program of table 30 of the EWWG report [22]. The numbers for A_e and A_τ in tables 4 and 5 agree with each other, while they disagree in the EWWG report in tables 30 and 31. In order to restore the agreement one has to interchange A_e and A_τ in table 30 of the EWWG report.

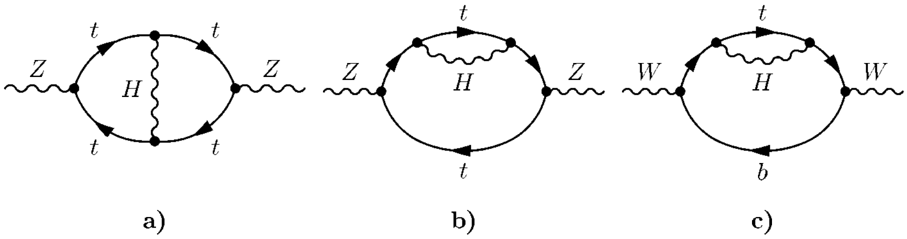

Figure 12. Some Feynman diagrams that give $\alpha_W^2 t^2$ corrections.

Finally, taking into account the definition of V_R, equations (49), (52), and equations (F.15), (F.7) we get:

$$g_{Vl}/g_{Al} = 1 - 4\left[1 - c^2 - \frac{c^2 s^2}{c^2 - s^2}\delta + \frac{c^4 s^4}{(c^2 - s^2)^3}\delta^2\right]$$
$$= 1 - 4s^2 + \frac{4c^2 s^2}{c^2 - s^2}\delta - \frac{4c^4 s^4}{(c^2 - s^2)^3}\delta^2, \qquad (99)$$

$$\delta_4' V_R = -\frac{3\bar{\alpha} t^2}{16\pi(c^2 - s^2)^2}. \qquad (100)$$

Formulae (96), (98) and (100) contain corrections to the functions V_i which come from the squares of polarization operators and are proportional to $\bar{\alpha} t^2$—so, it is leading ($\sim t^2$) parts of $(\Pi_Z - \Pi_W)^2$ corrections. Numerically they are several times smaller than $\bar{\alpha} t^2$ corrections which originate from irreducible diagrams.

7.2. $\alpha_W^2 t^2$ corrections from irreducible diagrams

The major part of the $\alpha_W^2 t^2$ corrections comes from the irreducible two-loop Feynman diagrams [56, 57]. The key observation in performing their calculation is that these corrections are of the order of $[\alpha_W (\frac{m_t}{m_Z})^2]^2 \sim \lambda_t^4$, where λ_t is the coupling constant of the higgs doublet with the top quark. That is why they can be calculated in a theory without vector bosons, taking into account only top–higgs interactions [56]. Corresponding pieces of vector boson self-energies can be extracted from the self-energies of would-be Goldstone bosons which enter the Higgs doublet (those components which, after mixing with massless vector bosons, form massive W and Z bosons). Correction of the order of $\alpha_W^2 t^2$ is contained in the difference $\Pi_Z(0) - \Pi_W(0)$ (see figure 12), so it is universal, i.e. one and the same for V_m, V_A and V_R. In [42] we call these corrections $\delta_4 V_i$:

$$\delta_4 V_i(t, h) = -\frac{\bar{\alpha}}{16\pi s^2 c^2} A(h/t) \cdot t^2, \qquad (101)$$

where function $A(h/t)$ is given in table 6. To obtain this table for $m_H/m_t < 4$ we use a table from [57], and for $m_H/m_t > 4$ we use expansion over m_t/m_H from [56]. For $m_t = 175$ GeV and $m_H = 150$ GeV we get $A = 6.4$ and $\delta_4 V_i(t, h) = -0.08$. This corresponds to the shifts: -12 MeV for m_W, 7×10^{-5} for s_l^2 and 5×10^{-5} for g_{Al}. One should compare these shifts with one-loop results: $\delta_{1\text{loop}} m_W = 400$ MeV, $\delta_{1\text{loop}} s_l^2 = 50 \times 10^{-5}$ and $\delta_{1\text{loop}} g_A = 100 \times 10^{-5}$. Recall that present experimental accuracy in m_W is 64 MeV, in s_l^2 is 20×10^{-5} and in g_{Al} is 30×10^{-5}.

There is one more place from which corrections $\sim\alpha_W^2 t^2$ appear: this is the $Z \to b\bar{b}$ decay. At one electroweak loop the t quark can propagate in the vertex triangle $(t\bar{t}W)$ (see section 5).

Table 6. Functions $A(m_H/m_t)$ and $\tau_b^{(2)}(m_H/m_t)$.

m_H/m_t	$A(m_H/m_t)$	$\tau_b^{(2)}(m_H/m_t)$	m_H/m_t	$A(m_H/m_t)$	$\tau_b^{(2)}(m_H/m_t)$
0.00	0.739	5.710	2.60	10.358	1.661
0.10	1.821	4.671	2.70	10.473	1.730
0.20	2.704	3.901	2.80	10.581	1.801
0.30	3.462	3.304	2.90	10.683	1.875
0.40	4.127	2.834	3.00	10.777	1.951
0.50	4.720	2.461	3.10	10.866	2.029
0.60	5.254	2.163	3.20	10.949	2.109
0.70	5.737	1.924	3.30	11.026	2.190
0.80	6.179	1.735	3.40	11.098	2.272
0.90	6.583	1.586	3.50	11.165	2.356
1.00	6.956	1.470	3.60	11.228	2.441
1.10	7.299	1.382	3.70	11.286	2.526
1.20	7.617	1.317	3.80	11.340	2.613
1.30	7.912	1.272	3.90	11.390	2.700
1.40	8.186	1.245	4.00	11.396	2.788
1.50	8.441	1.232	4.10	11.442	2.921
1.60	8.679	1.232	4.20	11.484	3.007
1.70	8.902	1.243	4.30	11.523	3.094
1.80	9.109	1.264	4.40	11.558	3.181
1.90	9.303	1.293	4.50	11.590	3.268
2.00	9.485	1.330	4.60	11.618	3.356
2.10	9.655	1.373	4.70	11.644	3.445
2.20	9.815	1.421	4.80	11.667	3.533
2.30	9.964	1.475	4.90	11.687	3.622
2.40	10.104	1.533	5.00	11.704	3.710
2.50	10.235	1.595			

That is why at two loops correction of the order $\alpha_W^2 t^2$ emerges. Due to this correction, functions $V_{Ab}(t,h)$ and $V_{Rb}(t,h)$ differ from the corresponding functions describing $Z \to d\bar{d}$ decay:

$$V_{Ab}(t,h) = V_{Ad}(t,h) - \frac{8s^2c^2}{3(3-2s^2)}(\phi(t) + \delta\phi(t,h)), \tag{102}$$

$$V_{Rb}(t,h) = V_{Rd}(t,h) - \frac{4s^2(c^2-s^2)}{3(3-2s^2)}(\phi(t) + \delta\phi(t,h)), \tag{103}$$

where function $\phi(t)$ was discussed in section 5 and

$$\delta\phi(t,h) = \delta_{\alpha_s}(t)\phi + \delta_H\phi(t,h)$$
$$= \frac{3-2s^2}{2s^2c^2}\left\{-\frac{\pi^2}{3}\left(\frac{\hat{\alpha}_s(m_t)}{\pi}\right)t + \frac{1}{16s^2c^2}\left(\frac{\bar{\alpha}}{\pi}\right)t^2\tau_b^{(2)}\left(\frac{h}{t}\right)\right\}. \tag{104}$$

The first term in curly braces, $\delta_{\alpha_s}\phi$, was taken into account earlier, see section 5, and the new correction $\delta_H\phi(t,h)$ is proportional to function $\tau_b^{(2)}(h/t)$. Function $\tau_b^{(2)}$ is given in table 6. To obtain this table for $m_H/m_t < 4$ we use a table from [57], and for $m_H/m_t > 4$ we use expansion over m_t/m_H from [56] in full analogy with function $A(h/t)$.

For $m_t = 175$ GeV, $m_H = 150$ GeV we have $\tau_b^{(2)} = 1.6$.

The change of Γ_b due to $\tau_b^{(2)} = 1.6$ equals 0.03 MeV, which corresponds to 2×10^{-5} shift in R_b, while experimental accuracy in R_b is 7×10^{-4} (the one-loop electroweak correction in R_b is -3.9×10^{-3}). The influence of $\tau_b^{(2)}$ on A_{FB}^b and A_b is even smaller (by a few orders of magnitude).

7.3. $\alpha_W^2 t$ corrections and the two-loop fit of experimental data

Corrections of the order $\alpha_W^2 t$ originate from the top loop contribution to W and Z boson self-energies with higgs or vector boson propagating inside the loop and are of the order of $g^2\lambda_t^2$. We take into account these corrections in our code LEPTOP using results of [58–60].

Before we present results of the electroweak precision data fit which take into account α_W^2 corrections, described in this section, we must discuss how good the approximation which takes into account $\alpha_W^2 t^2$ and $\alpha_W^2 t$ terms but neglects (still not calculated) α_W^2 terms should be. For $m_t = 175$ GeV we obtain $t \simeq 3.7$, thus at first glance we have good expansion parameter so that α_W^2 terms could be safely neglected. To check this let us consider first the one electroweak loop, where the enhanced $\alpha_W t$ terms can be compared with non-enhanced α_W terms.

By using equations (47) and (49) and by comparing them with experimental data one sees that for m_W/m_Z the $\alpha_W t$ term is equal to 0.0057, while the α_W term is -0.0014. As for g_{Vl}/g_{Al}, the two terms are 0.0122 and -0.0142. Thus for m_W/m_Z the $\alpha_W t$ term dominates, while for g_{Vl}/g_{Al} it is practically cancelled by the α_W term.

Returning to two-loop corrections we observe that the $\alpha_W^2 t^2$ correction to m_W is not larger than the $\alpha_W^2 t$ correction; for $m_t = 175$ GeV and $m_H = 150$ GeV it diminishes m_W by 23 MeV (compare with section 7.2).

In table 7 we present results of the fit of the data where we use theoretical formulae which include the two-loop electroweak corrections described in this section. Comparing table 7 with table 4 where the fit of the one-loop electroweak corrected formulae was presented, we see that the fitted values of all physical observables are practically the same, with one (very important) exception: the central value of the higgs mass becomes $\sim$70 GeV lower. In view of the previous discussion it seems reasonable to consider this shift as a cautious estimate of the theoretical uncertainty in m_H.

We have a simple qualitative explanation as to why $\alpha_W^2 t$ corrections reduce the higgs mass by $\sim$70 GeV. The point is that these corrections shift the theoretical value of s_l^2 by +0.0002, which is close to experimental error in s_l^2. In order to compensate for the shift, the fitted mass of the higgs changes. This change can be easily derived. Indeed, from equations (49), (52), (45) we get:

$$\delta s^2 = -\frac{3}{16\pi(c^2 - s^2)}\bar{\alpha}\delta H_R = -0.000\,86\delta H_R, \tag{105}$$

while from table 3 we see that changing m_H from 150 to 100 GeV gives $\delta H_R = +0.236$ and $\delta s^2 = -0.0002$.

In figure 13 the dependence of χ^2 on the value of higgs mass is shown both with and without inclusion of SLD data (Z-decays into heavy quark pairs are taken into account in both plots). When all existing data are taken into account we get a central value of higgs mass $m_H = 71$ GeV which is 20 GeV below the lower bound [22] of the LEP-II direct searches, $m_H > 95$ GeV. However, uncertainty in the value of m_H extracted from radiative corrections is quite large, thus there is no contradiction between these two numbers.

At the end of this section we would like to make two remarks demonstrating that one should not take too seriously the central values of m_H extracted from the global fits.

First, if one disregards the FNAL measurements of m_t, then one obtains from the fit:

$$m_t = 160.7^{+7.7}_{-6.8}\ \text{GeV}, \qquad m_H = 30.3^{+38.8}_{-14.4}\ \text{GeV}.$$

Such a value of m_H is 1.5 standard deviations below the lower bound from direct searches of LEP-II. (Note also that the fitted value of the top mass m_t is substantially lower than measured at FNAL).

Second, as was stressed in [61], the values of s_l^2 extracted from different observables lead to very different central values of m_H. For example, from SLAC data on A_{LR} it follows that

Table 7. Fit of experimental data [22] with two-electroweak-loop formulae. $m_Z = 91.1867(21)$ GeV is used as an input. Output of the fit: $m_H = 70.8^{+82}_{-43}$ GeV, $\hat{\alpha}_s = 0.1194 \pm 0.0029$, $\chi^2/n_{d.o.f.} = 15.0/14$. The most optimistic errors on M_H are obtained in the fit including $\bar{\alpha}(DH)^{-1} = 128.923(36)$ [30] and $\alpha_s(PDG) = 0.1178(23)$ from low-energy data [27]. Such a fit gives $m_H = 93^{+63}_{-41}$ GeV, $m_t = 171.3 \pm 4.8$ GeV, $\alpha_s = 0.1184 \pm 0.0018$, $\chi^2/n_{d.o.f.} = 15.2/14$. However the systematic errors due to the model assumptions used in the calculations of $\alpha_s(PDG)$ and $\bar{\alpha}(DH)$ are not easy to estimate. That is why we prefer to use the result with less optimistic assumptions leading to bigger error in m_H.

Observable	Experimental data	Standard model	Pull
(a) LEP-I			
Shape of Z-peak and lepton asymmetries:			
Γ_Z (GeV)	2.4939(24)	2.4960(18)	−0.9
σ_h (nb)	41.491(58)	41.472(16)	0.3
R_l	20.765(26)	20.746(20)	0.7
A^l_{FB}	0.0168(10)	0.0161(4)	0.7
τ-polarization:			
A_τ	0.1431(45)	0.1467(16)	−0.8
A_e	0.1479(51)	0.1467(16)	0.2
Results for heavy quarks:			
R_b^{a}	0.2166(7)	0.2158(2)	1.0
R_c^{a}	0.1735(44)	0.1723(1)	0.3
A^b_{FB}	0.0990(21)	0.1028(12)	−1.8
A^c_{FB}	0.0709(44)	0.0734(9)	−0.6
Charge asymmetry for pairs of light quarks $q\bar{q}$:			
$s_l^2(Q_{FB})$	0.2321(10)	0.2316(2)	0.5
(b) SLC			
$s_l^2\ (A_{LR})$	*0.2311(3)*	*0.2316(2)*	−1.6
A_{LR}	0.1504(23)	0.1467(16)	1.6
R_b^{a}	0.2166(7)	0.2158(2)	0.9
R_c^{a}	0.1735(44)	0.1723(1)	0.3
A_b	0.8670(350)	0.9348(2)	−1.9
A_c	0.6470(400)	0.6677(7)	−0.5
(c) $p\bar{p}$+ LEP-II $+\nu N$			
m_W (GeV) ($p\bar{p}$+ LEP-II)	80.3902(64)	80.3659(34)	0.4
	0.2228(13)		
$s_W^2\ (\nu N)$	0.2254(21)	0.2233(7)	1.0
	80.255(109)		
m_t (GeV)	173.8(5.0)	170.8(4.9)	0.6

^{rma} Experimental values of R_b and R_c correspond to the average of LEP-I and SLC results.

$m_H = 25$ GeV with a 90% confidence interval of 6–100 GeV. Even smaller values of m_H follow from LEP measurement of A^τ_{FB}: $m_H = 4$ GeV (0.2 GeV $< m_H <$ 95 GeV at 90% CL). As for other asymmetries measured at LEP, they lead to much heavier higgs: from A^b_{FB}, for example, $m_H = 370$ GeV (100 GeV $< m_H <$ 1400 GeV at 90% CL). That is why the average of all these values of m_H seems to be not very reliable.

As can be seen from table 8, the `LEPTOP` fit is very close to the `ZFITTER` fit [22] and to the fit by Erler and Langacker [23]. This indicates that theoretical uncertainties are very small, except for the non-calculated part of the corrections, which is common to all three programs.

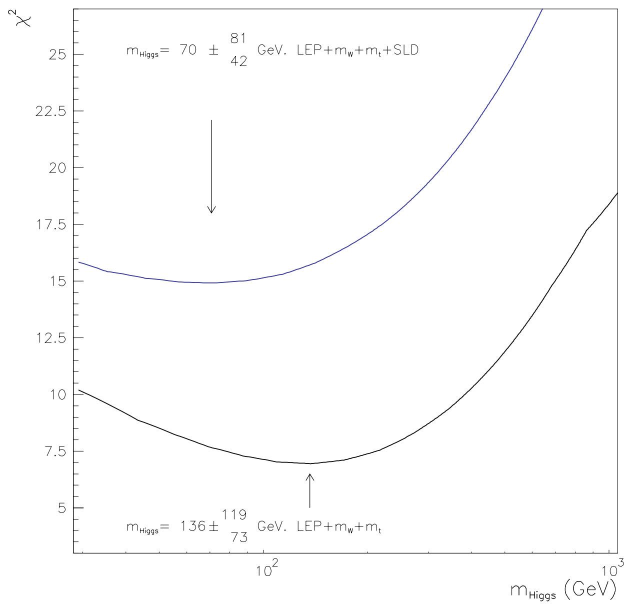

Figure 13. χ^2 versus m_H curves.

8. Extensions of the SM

The SM works well at the energy scale of the order of the vector bosons masses. We see that the SM description of the electroweak observables in this energy region is in perfect agreement with the precision measurements.

However there are many natural physical questions that have no satisfactory answers within the framework of the SM. So it is hard to believe that the SM is the final theory. The common expectation is that there should be new physics beyond the SM.

Direct accelerator searches have not yet found any trace of new physics. Their negative results have given lower bounds on the masses and upper bounds on the production cross sections for the new particles. In this section we are going to study the indirect bounds on new physics that can be theoretically derived from the precision measurements at low energy of the order of Z and W boson masses. Loops with hypothetical new particles change the predictions of the SM for electroweak observables. Since the SM gives a very good description of the data there is little room for such new contributions. In this way one can derive some constraints on new theory.

Any possible generalizations of the SM are naturally divided into two classes: theories with and without decoupling. In the first class, the contribution of new particles into W and Z boson parameters are suppressed as positive powers of $(m_Z^2/m^2)^n$ when the masses of new particles m become larger than electroweak scale. One cannot exclude such theory by studying loop corrections to low-energy observables. In this way one may hope to bound the masses of new particles from below. The most well known examples of such theory are supersymmetric extensions of the SM.

In the second class of theories the contribution of new particles into low-energy observables does not decouple even when their masses become very large. Such SM generalizations can be excluded if the additional nondecoupled contributions exceed the discrepancy between the SM fit and experimental data. An example of such generalization is the SM with additional sequential generations of quarks and leptons.

Table 8. Comparison of the LEPTOP, ZFITTER [22] and Erler–Langacker [23] fits. Erler–Langacker use slightly different experimental dataset for their fit. This may cause some of the discrepancies with LEPTOP and ZFITTER.

Observable	Experimental data	LEPTOP	EWWG ZFITTER	Erler–Langacker
(*a*) LEP-I				
M_Z (GeV)	91.1867(21)	91.1867 fix.	91.1865	91.1865(21)
Γ_Z (GeV)	2.4939(24)	2.4960(18)	2.4958	2.4957(17)
σ_h (nb)	41.491(58)	41.472(16)	41.473	41.473(15)
R_l	20.765(26)	20.746(20)	20.748	20.748(19)
A^l_{FB}	0.0168(10)	0.0161(4)	0.01613	0.0161(3)
A_τ	0.1431(45)	0.1467(16)	0.1467	0.1466(15)
A_e	0.1479(51)	0.1467(16)	0.1467	0.1466(13)
R_b	0.2166(7)	0.2158(2)	0.2159	0.2158(2)
R_c	0.1735(44)	0.1723(1)	0.1722	0.1723(1)
A^b_{FB}	0.0990(21)	0.1028(12)	0.1028	0.1028(10)
A^c_{FB}	0.0709(44)	0.0734(9)	0.0734	0.0734(8)
$s^2_l(Q_{FB})$	0.2321(10)	0.2316(2)	0.23157	0.2316(2)
(*b*) SLC				
$s^2_l(A_{LR})$	0.2311(3)	0.2316(2)	0.23157	—
A_{LR}	0.1504(23)	0.1467(16)	—	0.1466(15)
A_b	0.8670(350)	0.9348(2)	0.935	0.9347(1)
A_c	0.6470(400)	0.6677(7)	0.668	0.6676(6)
(*c*) $p\bar{p}$+ LEP-II +νN				
m_W (GeV) ($p\bar{p}$+ LEP-II)	80.3902(64)	80.3659(34)	80.37	80.362(23)
	0.2228(13)			
s^2_W (νN)	0.2254(21)	0.2233(7)	0.2232	
	80.255(109)			
m_t (GeV)	173.8(5.0)	170.8(4.9)	171.1(4.9)	171.4(4.8)
m_H (GeV)		71.0^{+82}_{-43}	76.0^{+85}_{-47}	107.0^{+67}_{-45}
α_s		0.1194(29)	0.119(3)	0.1206(30)
$\bar{\alpha}^{-1}$	128.878(90)	128.875	128.878	

8.1. Sequential heavy generations in the SM

We start the discussion of new physics with the simplest extension of the SM, namely the SM with additional sequential generations of leptons and quarks [62–64]. Nobody knows any deep reason for the number of generations to be equal to three. So it is interesting to study whether it is allowed to have four and more generations. Certainly these new generations should be heavy enough not to be produced in the Z decays and at LEP-II.

We consider the case of no mixing between the known generations and the new ones. In this case the new fermion generations effect the ratio m_W/m_Z and the widths and the decay asymmetries of the Z boson only through the vector bosons self-energies. Such corrections have been dubbed [88] 'oblique corrections'. We start their study with the case of $SU(2)$ degenerate fourth generation:

$$m_U = m_D = m_Q, \qquad m_N = m_E = m_L. \tag{106}$$

New terms in the self-energies modify the functions V_m, V_A, V_R, i.e. the radiative corrections to m_W/m_Z, g_{Al} and g_{Vl}/g_{Al}. The contribution to V_i from the fourth generation can be written in the form:

$$V_m \to V_m + \delta^4 V_m, \qquad V_A \to V_A + \delta^4 V_A, \qquad V_R \to V_R + \delta^4 V_R. \tag{107}$$

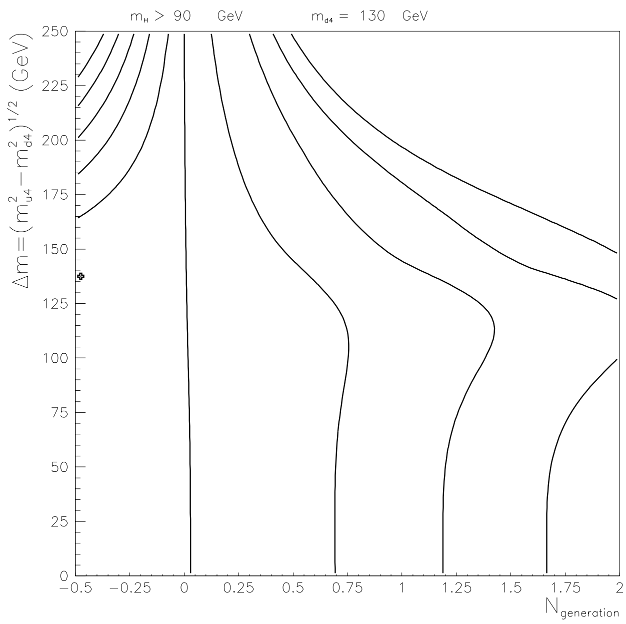

Figure 14. The two-dimensional exclusion plot for the case of N extra generations and for the choice $m_D = 130$ GeV—the lowest allowed value for new quark mass from Tevatron search [27], using $m_H > 90$ GeV at 95 % CL from LEP-2 [89]. The cross corresponds to χ^2 minimum; curves show one sigma, two sigma, etc allowed domains.

The analytical expressions for $\delta^4 V_i$ for quark or lepton doublets (neglecting gluonic corrections) can be found in appendix G, equations (G.1)–(G.3).

In the limit of a very heavy fourth generation of leptons and quarks one has:

$$\Sigma\delta^4 V_m \to -\tfrac{16}{9}s^2, \qquad \Sigma\delta^4 V_R \to -\tfrac{8}{9}, \qquad \Sigma\delta^4 V_A \to 0, \tag{108}$$

where Σ denotes sum over leptons and quarks with $m_Q = m_L = m_4$, $s^2 \simeq 0.23$. Equations (108) reflect the non-decoupling of the heavy degrees of freedom in electroweak theory, caused by the axial current. It is interesting that the contribution of degenerate generation to V_m, V_R has negative sign.

The fourth generation with strong violation of $SU(2)$ symmetry (i.e. with very large mass difference in the doublet) gives a universal contribution to functions $\delta^4 V_i$ (similar to the universal contribution of t and b quarks from the third generation to V_i):

$$\delta^4 V_i = 4|m_T^2 - m_B^2|/3m_Z^2. \tag{109}$$

In the case of large mass splitting, $\delta^4 V_i$ are positive. From equations (108) and (109) it is clear that somewhere in the intermediate region of mass splitting the functions $\delta^4 V_m$ and $\delta^4 V_R$ intersect zero. In the vicinities of these zeros the contribution of new generation to these specific observables is negligible and one cannot exclude these regions of masses by studying only one of the observables. Fortunately, for different observables these zeros are located in different places and the general fit overcomes such a conspiracy of new physics.

For different up and down quark (and lepton) masses analytical expressions for $\delta^4 V_i$ are given in appendix G, equations (G.4)–(G.6).

Figure 14 shows the two-dimensional exclusion plot for the case of n extra generations, where n is formally considered as a continuous parameter. We see from this plot that at 90%

CL we have less than one extra generation and at 99% CL, less than two extra generations, for any differences of up and down quark masses.

8.2. SUSY extensions of the SM

In this section we consider another example of new physics: supersymmetric extensions of the SM. There are certain aesthetic and conceptual merits of such SUSY generalization of the SM. Here are some of them:

(1) Supersymmetry gives a solution for the problem of fine tuning, i.e. it prevents the electroweak scale of the SM from mixing with the Planck scale.
(2) The problem of unification of electroweak and strong coupling constants seems to have solution in the framework of SUSY extensions.
(3) Finally, any ambitious 'theory of everything' inevitably includes SUSY as the basic element of the construction.

To give a systematic introduction to SUSY extensions of the SM would need a separate review paper (see, e.g., [65]). Here we are going to make a short sketch of this well developed branch of physics in applications to the theory of the Z boson. To construct SUSY extensions one has to introduce a lot of new particles. For example, minimal $N = 1$ supersymmetry automatically doubles the number of degrees of freedom of the SM: any fermionic degree of freedom has to be coupled with bosonic degree of freedom and vice versa. Thus the left(right) leptons have to be accompanied by scalars: 'left'('right') sleptons, quarks by squarks, gauge bosons by spinor particles—gauginos, etc. The Higgs mechanism of mass generation for up and down quarks requires two Higgs boson doublets (and two higgsino doublets, respectively).

Not one of these numerous new particles has been observed yet. If they do exist they are too heavy to be produced at the working accelerators. On the other hand, these heavy supersymmetric particles (again, if they do exist) are produced in the virtual states, i.e. in the loops. Loops with new particles change the predictions of the SM for the low-energy observables. (By 'low energy' we mean here $E \lesssim m_Z$.) In this indirect way one can get some information about the existence or nonexistence of SUSY.

The SUSY extensions of the SM belong to the class of new physics that decouples from the low-energy observables when the mass scale of this new physics becomes very large. This means that the additional contribution into electroweak observables due to the supersymmetric particles are of the order of $\alpha_W(m_W/m_{SUSY})^2$ or $\alpha_W(m_t/m_{SUSY})^2$, where m_{SUSY} characterizes the mass scale of superpartners. Since the fit of the precision data in the framework of the SM statistically is very good these new additional contributions have to be small. So in this way one expects to get strong restrictions on the value of m_{SUSY}.

Supersymmetric contributions into low-energy observables were studied in [66–69]. The results depend on the model and on the pattern of SUSY violation. Within a given model the results for low-energy observables are formulated in terms of the functions that depend on the fundamental parameters of the SUSY Lagrangian that are fixed at the high-energy scale of SUSY violation. The fit of experimental data in the framework of a given SUSY model imposes certain restrictions on the allowed region of these high-energy scale parameters of the model. As for the masses of sparticles, their values are calculated by numerical solution of the renormalization group equations. They also depend on the fundamental SUSY parameters at the high-energy scale. In this rather indirect way one gets restrictions on the physical masses of sparticles in the general case.

To give the reader a taste of the exploration of the new supersymmetric physics we consider in this section only that part of the multi-dimensional space of SUSY parameters for which all sparticles have more or less the same masses, i.e. when we have no light sparticles. (It seems reasonable to start the study of the unknown field with one of the simplest assumptions). In this case one can find the class of enhanced oblique corrections which are universal, i.e. that are the same for any model. Another merit of these corrections is that they directly depend on the masses of sparticles.

As will be shown, the enhanced electroweak radiative SUSY corrections are induced by the large violation of $SU(2)_L$ symmetry in the third generation of squarks. Therefore we start the discussion of the SUSY corrections to the functions V_i with the brief description of the stop $(\tilde{t}_L, \tilde{t}_R)$ and sbottom $(\tilde{b}_L, \tilde{b}_R)$ sector of the theory. The following relation between masses of quarks q and diagonal masses of left squarks $\tilde{q}_L$ takes place in a wide class of SUSY models:

$$m^2_{\tilde{q}_L} = m_q^2 + m^2_{SUSY} + m_Z^2 \cos(2\beta)(T_3 - s^2 Q_q), \tag{110}$$

where $s^2 \simeq 0.23$, Q_q is the charge and T_3 is the third projection of weak isospin of quark and tgβ is equal to the ratio of the VEV of two Higgs fields, introduced in SUSY models. The second term in the r.h.s. of equation (110) violates supersymmetry. It is some universal $SU(2)$-blind SUSY-violating soft mass term. The third term in r.h.s. of equation (110) also violates SUSY. It originates from the quartic D term in the effective potential and is different for up and $down$ components of the doublets. The only hypothesis that is behind this relation is that the origin of the large breaking of this $SU(2)_L$ is in the quark–higgs interaction.

Therefore, from equation (110) we get the following relation between masses of stop $m^2_{\tilde{t}_L}$, of sbottom $m^2_{\tilde{b}_L}$ and of top m_t^2 (we neglect m_b):

$$m^2_{\tilde{t}_L} - m^2_{\tilde{b}_L} = m_t^2 + m_Z^2 \cos(2\beta) c^2. \tag{111}$$

Relation (111) is central to this approach. It demonstrates the large violation of $SU(2)_L$ symmetry in the third generation of squarks. On the other hand, it demonstrates that in the limit of very large mass the left stop and left sbottom become degenerate and the parameter $(m^2_{\tilde{t}_L} - m^2_{\tilde{b}_L})/m^2_{\tilde{b}_L}$ goes to zero when m_{SUSY} goes to infinity. That is why the physical observables can depend on this decoupling parameter.

As for the right sparticles from the third generation, they are $SU(2)_L$ singlets. But they can mix with the left sparticles and in this way they contribute into enhanced corrections. The mixing between $\tilde{b}_L, \tilde{b}_R$ has to be proportional to m_b and can be neglected. The $\tilde{t}_L \tilde{t}_R$ mass matrix in general has the following form:

$$\begin{pmatrix} m^2_{\tilde{t}_L} & m_t A'_t \\ m_t A'_t & m^2_{\tilde{t}_R} \end{pmatrix}, \tag{112}$$

where $\tilde{t}_L \tilde{t}_R$ mixing is proportional to m_t and therefore is not small. Coefficient A'_t depends on the model. Diagonalizing matrix (112) we get the following eigenstates:

$$\begin{aligned} \tilde{t}_1 &= c_u \tilde{t}_L + s_u \tilde{t}_R \\ \tilde{t}_2 &= -s_u \tilde{t}_L + c_u \tilde{t}_R, \end{aligned} \tag{113}$$

where $c_u \equiv \cos\theta_{LR}$, $s_u \equiv \sin\theta_{LR}$, θ_{LR} is the $\tilde{t}_L \tilde{t}_R$ mixing angle, and

$$\text{tg}^2\theta_{LR} = \frac{m_1^2 - m^2_{\tilde{t}_L}}{m^2_{\tilde{t}_L} - m_2^2}, \qquad m_1^2 \geqslant m^2_{\tilde{t}_L} \geqslant m_2^2. \tag{114}$$

Parameters m_1 and m_2 are the mass eigenvalues:

$$m^2_{1,2} = \frac{m^2_{\tilde{t}_L} + m^2_{\tilde{t}_R}}{2} \pm \frac{|m^2_{\tilde{t}_L} - m^2_{\tilde{t}_R}|}{2} \sqrt{1 + \frac{4m_t^2 A'^2_t}{(m^2_{\tilde{t}_L} - m^2_{\tilde{t}_R})^2}}. \tag{115}$$

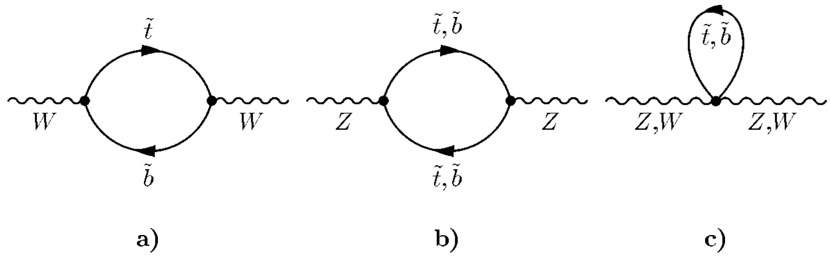

Figure 15. Contribution of $\tilde{t}$ and $\tilde{b}$ squarks into W and Z bosons self-energy.

The enhanced electroweak radiative corrections are induced by the contribution of the third generation of squarks into self-energy operators of vector bosons. Nondiagonal vector currents of squarks are not conserved only because of violation of $SU(2)$ by mass terms. Thus one should expect that the self-energy operators are proportional to the divergency of the currents. To calculate these enhanced terms it is sufficient to expand the operators of the vector bosons $\Sigma_V(k^2)$ at $k^2 = 0$. The terms enhanced as m_t^4/M_{SUSY}^2 come from $\Sigma_W(0)$, while those enhanced as $m_t^2 M_W^2/M_{SUSY}^2$ come from $\Sigma'_{W,Z}(0)$ (see figure 15). These simple self-energy corrections are obviously universal since stop and sbottom should exist in any SUSY model and the coupling constants are universal since they are fixed by gauge invariance only. The higher-order derivatives of self-energies are suppressed as $(m_{W,Z}/m_{SUSY})^2$. They are of the same order of magnitude as the numerous model-dependent terms coming from vertex and box diagrams. If there are no very light sparticles, the first two universal terms have rather large enhancement factor of the order of $t^2 \simeq 14$ and $t \simeq 3.7$, respectively. (The presence of terms $\sim m_t^4$ in SUSY models was recognized long ago [70].) We neglect the non-enhanced terms. The accuracy of such approximation may be of the order of 10% if we are lucky, but it may be as well of the order of unity (see the discussion of V_R in section 5 and of the two-loop corrections in section 7). As for the stop contributions to the vertex corrections, there is only one relevant case—the amplitude of $Z \to b\bar{b}$ decay. For vertex with stop exchange there are no terms enhanced as $(m_t/m_W)^4$ [71]. Thus we will neglect the corresponding corrections as well.

The calculation of the two enhanced terms is a rather trivial exercise. The only subtle point is the diagonalization of the stop propagators. The result of calculations depends on three parameters: m_1, m_2 and $m_{\tilde{b}_L}$. The dependence on angle β is very moderate and in numerical fits we will use the rather popular value tg$\beta = 2$. In what follows, instead of $m_{\tilde{b}_L}$ we will write $m_{\tilde{b}}$, bearing in mind that $\tilde{b}_L\tilde{b}_R$ mixing is proportional to m_b and can be neglected. The formulae that describe the enhanced SUSY corrections to the functions V_i can be found in appendix G, equations (G.7)–(G.11).

There is also another source of the potentially large SUSY corrections: vertices with gluino exchange of the order $\hat{\alpha}_s(m_Z/m_{SUSY})^2$.

These corrections shift the radiators R_{V_q} and R_{A_q} in equation (31) [73]:

$$\delta R_{V_q} = \delta R_{A_q} = 1 + \frac{\hat{\alpha}_s(m_Z)}{\pi}\Delta_1(x, y), \qquad (116)$$

$$\Delta_1(x, y) = -\frac{4}{3}\int_0^1 \mathrm{d}z_1 \int_0^{1-z_1} \mathrm{d}z_2 \log\left[1 - \frac{xyz_1z_2}{x + (z_1 + z_2)(y - x)}\right], \qquad (117)$$

where $x = (m_Z/m_{\tilde{q}})^2$, $y = (m_Z/m_{\tilde{g}})^2$, and $\Delta_1(x, x) \simeq \frac{1}{18}x + \cdots$. We take these gluino corrections into account in our analysis. The electroweak SUSY corrections to g_{A_q} and g_{V_q} are generated by the corrections to the function V_A, equation (G.7), and V_R, equation (G.8).

Table 9. Fit of the precision data with SUSY corrections taken into account in the case of the absence of $\tilde{t}_L\tilde{t}_R$ mixing, $\sin\theta_{LR} = 0$ and m_h taken as a free parameter. For $m_{\tilde{b}} > 300$ GeV SUSY.

$m_{\tilde{b}}$ (GeV)	m_h (GeV)	$\hat{\alpha}_s$	$\chi^2/n_{d.o.f.}$
100	850^{+286}_{-320}	0.113 ± 0.003	20.3/14
150	484^{+364}_{-235}	0.116 ± 0.003	18.1/14
200	280^{+240}_{-144}	0.117 ± 0.003	17.3/14
300	152^{+145}_{-87}	0.118 ± 0.003	16.3/14
400	113^{+115}_{-68}	0.119 ± 0.003	15.8/14
1000	77^{+87}_{-47}	0.119 ± 0.003	15.2/14

Table 10. As table 9 but with the value of the lightest Higgs boson mass $m_h = 120$ GeV which is about the maximum allowed value in the simplest SUSY models.

$m_{\tilde{b}}$ (GeV)	$\hat{\alpha}_s$	$\chi^2/n_{d.o.f.}$
100	0.110 ± 0.003	30.2/15
150	0.115 ± 0.003	21.9/15
200	0.116 ± 0.003	18.6/15
300	0.118 ± 0.003	16.4/15
400	0.119 ± 0.003	15.8/15
1000	0.119 ± 0.003	15.5/15

Having all the necessary formulae in hand, we start the new fit of the data with the simplest case of the absence of $\tilde{t}_L\tilde{t}_R$ mixing, $\sin\theta_{LR} = 0$. In this case we have only one additional mass parameter. Thus we expect that this mass should be heavy enough not to destroy the perfect SM fit of the experimental data. First, let us take the value of the lightest neutral Higgs boson mass as a free parameter and take the masses of the other three Higgs bosons to be very heavy. The results of the fit are shown in table 9. We see that to fit the data with light sbottom one has to take the mass of the Higgs boson as much larger than its SM fit value. Even in this case the quality of the fit is worse than the SM one. For very heavy sbottom, one reproduces the SM fit. (To reduce the number of parameters we take $m_{\tilde{g}} = m_{\tilde{b}}$ in this fit. Let us stress that light squarks with masses of the order of 100–200 GeV are usually allowed only if gluinos are heavy, $m_{\tilde{g}} \geqslant 500$ GeV [74]. In the case of heavy gluino the correction Δ_1 (equation (117)) becomes power suppressed and we return to the SM fit value of $\hat{\alpha}_s = 0.119(3)$.)

We see that to get a reasonably good fit of the data in the framework of the SUSY extensions with light squarks one has to put the lightest higgs mass in the TeV region. Recall that in SUSY models the mass of the Higgs boson is not an absolutely free parameter. In the minimal supersymmetric standard model (MSSM), among three neutral Higgs bosons the lightest one should have mass less than approximately 120–135 GeV [76]. If other higgses are considerably heavier the lightest scalar boson has the same couplings with gauge bosons as in the SM. As a result, the same SM formulae for radiative corrections can be used in the SUSY extensions of the SM. (Deviations from the SM formulae are suppressed as $(m_h/m_A)^2$, where m_A is the mass of the heavier higgs. We will assume in our analysis that m_A is large). For the maximum allowed value, $m_h \simeq 120$ GeV, the results of the fit are shown in table 10. (In what follows we will always take $m_h \simeq 120$ GeV, since for 90 GeV $< m_h <$ 135 GeV the results of the fit are practically the same.) Table 10 demonstrates that superpartners should be heavy if we want to have a good fit of the data.

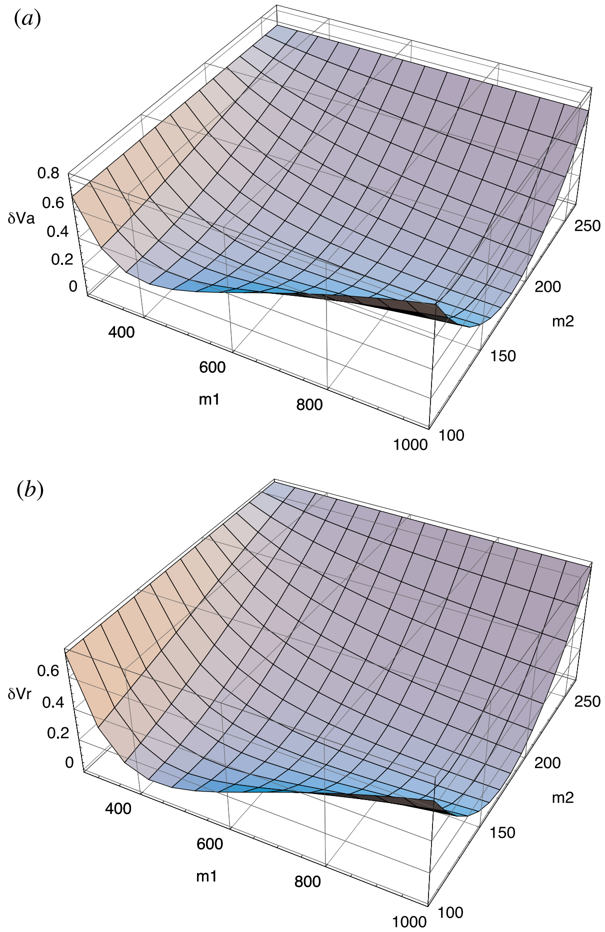

Figure 16. Values of δV_A, δV_R and δV_m at $m_{\tilde b} = 200$ GeV.

The next step is to take into account $\tilde t_L \tilde t_R$ mixing. In figure 16 we show the dependence of SUSY corrections $\delta_{SUSY} V_i$ on m_1 and m_2 for $m_{\tilde b} = 200$ GeV. One clearly sees from this figure that even for this small value of $m_{\tilde b}$ there exist the domain of low m_2 values where the enhanced radiative corrections are suppressed. In figure 16 one sees the valley where $\delta_{SUSY} V_i$ reaches its minimum values which are considerably smaller than one. The valley starts at $m_2 \simeq m_{\tilde b}$, $m_1 \simeq 1000$ GeV and goes to $m_2 \simeq 100$ GeV, $m_1 \simeq 400$ GeV. The smallness of the radiative corrections at the point $m_2 \simeq m_{\tilde b}$, $m_1 \simeq 1000$ GeV can be easily understood: here $\theta_{LR} \simeq \pi/2$, $\tilde t_2 \simeq \tilde t_L$, $\tilde t_1 \simeq \tilde t_R$. Thus nondiagonal charged left current of squarks is conserved and the main enhanced term vanishes. Indeed, in $\delta_{SUSY} V_A$ only the term proportional to $g(m_2, m_{\tilde b})$ remains in equation (G.7), but for $m_2 = m_{\tilde b}$ it is equal to zero. At this end point of the valley $\tilde t_2 \simeq \tilde t_L$, $\tilde t_1 \simeq \tilde t_R$, so $m^2_{\tilde t_R} \gg m^2_{\tilde t_L}$, which is opposite to the relation between $m_{\tilde t_R}$ and $m_{\tilde t_L}$ occurring in a wide class of models. In these models (e.g. in the MSSM) the left and the right squark masses are equal at the high-energy scale. When renormalizing them to low energies one gets $m^2_{\tilde t_L} > m^2_{\tilde t_R}$. Almost along the whole valley we have $\mathrm{tg}^2\theta_{LR} > 1$, which means that $m^2_{\tilde t_R} > m^2_{\tilde t_L}$. This possibility of suppressing radiative corrections was discussed in [75] . However, in the vicinity of the end point $m_1 \simeq 300$ GeV, $m_2 \simeq 70$ GeV the value of $\mathrm{tg}^2\theta_{LR}$ becomes smaller

(*c*)

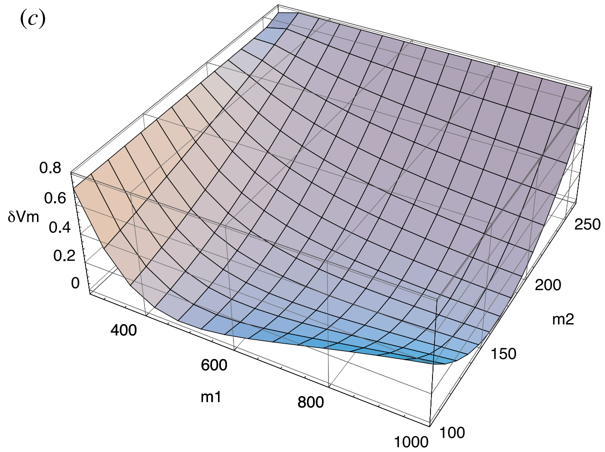

Figure 16. (Continued.)

Table 11. Results of fit along the valley of minimum of χ^2 for fixed value $m_{\tilde{b}} \simeq 200$ GeV and $m_h \simeq 120$ GeV.

m_1 (GeV)	m_2 (GeV)	$\hat{\alpha}_s$	$\chi^2/n_{d.o.f.}$
1296	193	0.118 ± 0.003	15.6/15
888	167	0.118 ± 0.003	15.8/15
387	131	0.118 ± 0.003	16.1/15
296	72	0.117 ± 0.003	16.7/15

than one and $m^2_{\tilde{t}_R} < m^2_{\tilde{t}_L}$.

In table 11 we show values of χ^2 along the valley of its minimum, which is formed for $m_{\tilde{b}} = 200$ GeV. We observe that a good fit is possible for light superpartners if $\tilde{t}_L\tilde{t}_R$ mixing is taken into account.

The main lesson of this section is the following. The fit of the precision data on electroweak observables (i.e. of Z boson decay parameters from LEP and SLC and the values of m_W and the m_t from Tevatron) in the framework of SUSY extension of the SM assuming small value of $m_{\tilde{b}}$, the absence of $\tilde{t}_L\tilde{t}_R$ mixing and $m_h = 120$ GeV leads to the growth of χ^2 value. For heavy squarks, the SUSY sector of the theory decouples from low-energy observables and the results of SM fit are reproduced. On the other hand, even for light sbottom and for small mass of one of two stops, one can find the values of $\tilde{t}_L\tilde{t}_R$ mixing where supersymmetric corrections appear to be small and not excluded by experimental data. In this case the quality of the fit (i.e. the value of χ^2) is almost the same as in the SM.

9. Conclusions

The comparison of LEP-I and SLC precision data on Z boson decays with calculations based on the MSM has confirmed the predictive power of the latter.

(1) It was proved that there exist only three generations of quarks and leptons with light neutrinos.
(2) The Z boson couplings of quarks, charged leptons and neutrinos are in accord with the theory.

Choose three observables measured with the highest accuracy:
$G_\mu, m_Z, \alpha(m_Z) \equiv \bar{\alpha}$

Determine angle θ ($s \equiv \sin\theta, c \equiv \cos\theta$)
in terms of $G_\mu, m_Z, \bar{\alpha}$: $G_\mu = (\pi/\sqrt{2})\bar{\alpha}/s^2c^2m_Z^2$

Introduce bare coupling constants in the framework of MSM
$(\alpha_0, \alpha_{Z0}, \alpha_{W0})$, bare masses $(m_{Z0}, m_{W0}, m_{H0}, m_{t0}, m_{q0})$
and the vacuum expectation value (VEV) of the higgs field, η.

Express $\alpha_0, \alpha_{Z0}, m_{Z0}$ in terms of $G_\mu, m_Z, \bar{\alpha}$ in one-loop
approximation, using dimensional regularization $(1/\varepsilon, \mu)$.

Express one-electroweak-loop corrections to all electroweak
observables in terms of $\alpha_0, \alpha_{Z0}, m_{Z0}, m_t, m_H$, and hence, in terms of
of $G_\mu, m_Z, \bar{\alpha}, m_t, m_H$. Check cancellation of the terms $(1/\varepsilon, \mu)$.

Introduce gluon corrections to quark loops and QED (and QCD)
final state interactions, as well as the two-electroweak-loop
corrections, calculated by other authors, in terms of
$\bar{\alpha}, \alpha_s(m_Z), m_H, m_t, m_b$.

Compare the predictions of the successive approximations
(Born, one loop, two loops) with the experimental data
on Z-decays. Perform the global fits for $m_H, m_t, \alpha_s(m_Z), \bar{\alpha}$
for one and two loops. Derive theoretical predictions
of the central values for all electroweak observables
and of the corresponding uncertainties (‘errors’).

Check the sensitivity of the fits to the possible
existence of new heavy particles: extra generations,
SUSY, technicolour, Z' etc.

(3) From the analysis of the radiative corrections the mass of the top quark had been correctly predicted before this particle was discovered at Tevatron.

(4) All electroweak observables (except for the mass of the higgs) are perfectly fitted by one-loop electroweak corrections (with virtual and 'free' gluons being taken into account).

(5) The dependence of the radiative corrections on the mass of the higgs is feeble when the higgs is heavy. Therefore the value of the higgs mass extracted from LEP-I and SLC data has rather large error bars. Within one standard deviation, the central fitted value of the higgs mass becomes smaller when the leading two-electroweak-loop corrections are taken into account. In this case it is close to 90 GeV—its direct lower limit from the LEP-II search. However the non-leading two-loop corrections may change this result. Calculation of these corrections is a challenge to theorists. Better understanding of the systematic discrepancies between various asymmetries in Z-decays is a challenge to experimentalists.

(6) One of the main conclusions of the one-electroweak-loop case is that in this case the value of the leading and non-leading corrections are comparable and even may cancel each other (in the case of leptonic parity violating parameter g_{Vl}/g_{Al}).

(7) The remarkable agreement between the MSM and experimental data on Z-decays puts strong limits on the hypothetical 'new physics', such as extra generations of heavy quarks and leptons and/or properties of supersymmetric particles.

The discovery of the higgs, more precise measurements of the mass of the W boson at LEP-II and Tevatron, and more accurate prediction of the value of electric charge at the scale of m_Z, may substantially improve the sensitivity of the Z-decay data to the possible manifestations of the new physics.

We conclude this review with a flowchart summarizing the theoretical approach used by us.

Acknowledgments

LBO is grateful to Alexander von Humboldt Stiftung for an award, ANR is grateful to CPPM/IN2P3/CNRS for supporting this work. The grants of RFBR No 96-15-96578, 98-07-90076, 98-02-17372 and 98-0217453 and of INTAS-RFBR 95-05678 are acknowledged. The authors would like to thank D Bardin, A Olshevski and P Renton for correspondence. We are grateful to our collaborators on various topics on Z boson decays.

Appendix A. Regularization of Feynman integrals

Integrals corresponding to diagrams with loops formally diverge and thus need regularization. Note that there does not yet exist a consistent regularization of electroweak theory in all loops. A dimensional regularization can be used in the first several loops; this corresponds to a transition to a D-dimensional spacetime in which the following finite expression is assigned to the diverging integrals:

$$\int \frac{\mathrm{d}^D p}{\mu^{D-4}} \frac{(p^2)^s}{(p^2+m^2)^\alpha} = \frac{\pi^{\frac{D}{2}}}{\Gamma(\frac{D}{2})} \frac{\Gamma(\frac{D}{2}+s)\Gamma(\alpha-\frac{D}{2}-s)}{\Gamma(\alpha)} \frac{(m^2)^{\frac{D}{2}-\alpha+s}}{\mu^{D-4}}, \qquad \text{(A.1)}$$

where μ is a parameter with mass dimension, introduced to conserve the dimension of the original integral.

This formula holds in the range of convergence of the integral. In the range of divergence, a formal expression (A.1) is interpreted as the analytical continuation. Obviously, the integral allows a shift in integration variable in the convergence range as well. Therefore, a shift

$p \to p+q$ for arbitrary D can also be done in (A.1). This factor is decisive in proving the gauge invariance of dimensional regularization.

At $D = 4$, the integrals in (A.1) contain a pole term

$$\Delta = \frac{2}{4-D} + \log 4\pi - \gamma - \log\frac{m^2}{\mu^2}, \tag{A.2}$$

where $\gamma = 0.577\ldots$ is the Euler constant. Choice of constant terms in (A.2) is a matter of convention.

The algebra of γ-matrices in the D-dimensional space is defined by the relations

$$\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2g_{\mu\nu} \times I, \tag{A.3}$$

$$g_{\mu\mu} = D, \tag{A.4}$$

$$\gamma_\mu\gamma_\nu\gamma_\mu = (2-D)\gamma_\nu, \tag{A.5}$$

where I is the identity matrix.

As for the dimensionality of spinors, different approaches can be chosen in the continuation to the D-dimensional space. One possibility is to assume that the γ-matrices are 4×4 matrices, so that

$$\mathrm{Sp} I = 4. \tag{A.6}$$

The D-dimensional regularization creates difficulties when one has to define the absolutely antisymmetric tensor and (or) γ_5 matrix. For calculations in several first loops, a formal definition of γ_5,

$$\gamma_5\gamma_\mu + \gamma_\mu\gamma_5 = 0, \tag{A.7}$$

$$\gamma_5^2 = I \tag{A.8}$$

does not lead to contradictions.

Thus, the amplitudes of physical processes, once they are expressed in terms of bare charges and bare masses, contain pole terms $\sim 1/(D-4)$.

If we eliminate bare quantities and express some physical observables in terms of other physical observables, then all pole terms cancel out. The general property of renormalizability guarantees this cancellation. (We have verified this cancellation directly in [33].) The 'five steps' described in section 4.6 are based on this renormalization procedure.

In order to avoid divergences in intermediate expressions, one can agree to subtract from each Feynman integral the pole terms $\sim 1/(4-D)$, since they will cancel out anyway in the final expressions. Depending on which constant terms (in addition to pole terms) are subtracted from the diagrams, different subtraction schemes arise: the $\overline{MS}$ scheme corresponds to subtracting the universal combination

$$\frac{2}{4-D} - \gamma + \log 4\pi.$$

Appendix B. Relation between $\bar{\alpha}$ and $\alpha(0)$

We begin with the following well known relation of quantum electrodynamics [77]:

$$\alpha(q^2) = \frac{\alpha(0)}{1 + \Sigma_\gamma(q^2)/q^2 - \Sigma'_\gamma(0)}. \tag{B.1}$$

Here the fine structure constant $\alpha \equiv \alpha(0)$ is a physical quantity. It can be measured as a residue of the Coulomb pole $1/q^2$ in the scattering amplitude of charged particles. As for the running coupling constant $\alpha(q^2)$, it can be measured from the scattering of particles with large masses m at low momentum transfer: $m \gg \sqrt{|q^2|}$. In the SM we have the Z boson, and the contribution of the photon cannot be identified unambiguously if $q^2 \neq 0$. Therefore, the definition of the running constant $\alpha(q^2)$ becomes dependent on convention and on details of calculations.

At $q^2 = m_Z^2$, the contribution of W bosons to $\bar{\alpha} \equiv \alpha(m_Z^2)$ is not large, so it is convenient to make use of the definition accepted in QED:

$$\bar{\alpha} = \frac{\alpha}{1 - \delta\alpha}, \tag{B.2}$$

where

$$\begin{aligned} \delta\alpha &= -\Pi_\gamma(m_Z^2) + \Sigma'_\gamma(0), \\ \Pi_\gamma(m_Z^2) &= \frac{1}{m_Z^2}\Sigma_\gamma(m_Z^2). \end{aligned} \tag{B.3}$$

The one-loop expression for the self-energy of the photon can be rewritten as [78]:

$$\begin{aligned} \Sigma_\gamma(s) = (\alpha/3\pi)\sum_f N_c^f Q_f^2 [s\Delta_f + (s + 2m_f^2)F(s, m_f, m_f) - s/3] \\ -(\alpha/4\pi)[3s\Delta_W + (3s + 4m_W^2)F(s, m_W, m_W)], \end{aligned} \tag{B.4}$$

where $s \equiv q^2$, the subscript f denotes fermions, the sum Σ_f runs through lepton and quark flavours, and N_c^f is the number of colours. The contribution of fermions to $\Sigma_\gamma(q^2)$ is independent of gauge. The last term in (B.4) refers to the gauge-dependent contribution of W bosons; the 't Hooft–Feynman gauge was used in equation (B.4).

The singular term Δ_i is:

$$\Delta_i = \frac{1}{\epsilon} - \gamma + \log 4\pi - \log\frac{m_i^2}{\mu^2}, \tag{B.5}$$

where $2\epsilon = 4 - D$ (D is the variable dimension of spacetime, $\epsilon \to 0$), $\gamma = -\Gamma'(1) = 0.577\ldots$ is the Euler constant and μ is an arbitrary parameter. Both $1/\epsilon$ and μ vanish in relations between observables.

The function $F(s, m_1, m_2)$ is defined by the contribution to self-energy of a scalar particle at $q^2 = s$, owing to a loop with two scalar particles (with masses m_1 and m_2) and with the coupling constant equal to unity:

$$F(s, m_1, m_2) = -1 + \frac{m_1^2 + m_2^2}{m_1^2 - m_2^2}\log\frac{m_1}{m_2} - \int_0^1 \mathrm{d}x \log\frac{x^2 s - x(s + m_1^2 - m_2^2) + m_1^2 - \mathrm{i}\epsilon}{m_1 m_2}. \tag{B.6}$$

The function F is normalized in such a way that it vanishes at $q^2 = 0$, which corresponds to subtracting the self-energy at $q^2 = 0$:

$$F(0, m_1, m_2) = 0. \tag{B.7}$$

The following formula holds for $m_1 = m_2 = m$:

$$F(s, m, m) \equiv F(\tau) = \begin{cases} 2\left[1 - \sqrt{4\tau - 1}\arcsin\dfrac{1}{\sqrt{4\tau}}\right], & 4\tau > 1, \\ 2\left[1 - \sqrt{1 - 4\tau}\log\dfrac{1 + \sqrt{1 - 4\tau}}{\sqrt{4\tau}}\right], & 4\tau < 1, \end{cases} \tag{B.8}$$

where $\tau = m^2/s$.

Let us present the following useful equality which holds for $F(\tau)$ derivative:

$$F'(\tau) \equiv -\tau\frac{\mathrm{d}}{\mathrm{d}\tau}F(\tau) = \frac{1 - 2\tau F(\tau)}{4\tau - 1}. \tag{B.9}$$

To calculate the contributions of light fermions, the t quark and the W boson to $\delta\alpha$, we need the asymptotics $F(\tau)$ for small and large τ:

$$F(\tau) \simeq \log\tau + 2 + \cdots, \qquad |\tau| \ll 1, \tag{B.10}$$

$$F(\tau) \simeq \frac{1}{6\tau} + \frac{1}{60\tau^2} + \cdots, \qquad |\tau| \gg 1. \tag{B.11}$$

As a result we obtain

$$\Pi_\gamma(m_Z^2) \equiv \frac{\Sigma_\gamma(m_Z^2)}{m_Z^2} = \frac{\alpha}{3\pi}\sum_8 N_c^f Q_f^2\left(\Delta_Z + \frac{5}{3}\right) + \frac{\alpha}{\pi}Q_f^2\left[\Delta_t + (1+2t)F(t) - \frac{1}{3}\right]$$
$$-\frac{\alpha}{4\pi}[3\Delta_W + (3+4c^2)F(c^2)], \tag{B.12}$$

where $t = m_t^2/m_Z^2$, and

$$\Sigma'_\gamma(0) = \frac{\alpha}{3\pi}\sum_9 N_c^f Q_f^2\Delta_f - \frac{\alpha}{4\pi}\left(3\Delta_W + \frac{2}{3}\right), \tag{B.13}$$

$$\delta\alpha = \frac{\alpha}{\pi}\left\{\sum_8 \frac{N_c^f Q_f^2}{3}\left(\log\frac{m_Z^2}{m_f^2} - \frac{5}{3}\right) - Q_t^2\left[(1+2t)F(t) - \frac{1}{3}\right] + \left[\left(\frac{3}{4} + c^2\right)F(c^2) - \frac{1}{6}\right]\right\}. \tag{B.14}$$

Therefore, $\delta\alpha$ is found as a sum of four terms,

$$\delta\alpha = \delta\alpha_l + \delta\alpha_h + \delta\alpha_t + \delta\alpha_W. \tag{B.15}$$

In the one-loop approximation:

$$\delta\alpha_l = \frac{\alpha}{3\pi}\sum_3\left[\log\frac{m_Z^2}{m_l^2} - \frac{5}{3}\right] = 0.031\,41. \tag{B.16}$$

Higher loops [28] give:

$$\delta\alpha_l = 0.031\,498. \tag{B.17}$$

Loops with top quarks give:

$$\delta\alpha_t \simeq -\frac{\alpha}{\pi}\frac{4}{45}\left(\frac{m_Z}{M_t}\right)^2 = -0.000\,05(1), \tag{B.18}$$

where we have used that $m_t = 175 \pm 10$ GeV. Note that $\delta\alpha_t$ is negligible and has the antiscreening sign (the screening of the t quark loops in QED begins at $q^2 \gg m_t^2$, while in our case $q^2 = m_Z^2 < m_t^2$).

Finally, the W-loop gives

$$\delta\alpha_W = \frac{\alpha}{2\pi}\left[(3+4c^2)\left(1 - \sqrt{4c^2-1}\arcsin\frac{1}{2c}\right) - \frac{1}{3}\right] = 0.000\,50. \tag{B.19}$$

The value of $\delta\alpha_W$ depends on gauge [79]; here we give the result of calculations in the 't Hooft–Feynman gauge. Traditionally, the definition of $\bar{\alpha}$ takes into account the contributions of leptons and five light quarks only. The terms $\delta\alpha_t$ and $\delta\alpha_W$ are taken into account in the electroweak radiative corrections. In our approach, these terms give the corrections $\delta_1 V_i$. In the same way the loops of not yet discovered heavy new particles ('new physics') should be accounted for.

Appendix C. How $\alpha_W(q^2)$ and $\alpha_Z(q^2)$ 'crawl'

The effect of 'running' of electromagnetic coupling constants $\alpha(q^2)$ (logarithmic dependence of the effective charge on momentum transfer q^2) has been known for more than four decades [77]. In contrast to $\alpha(q^2)$, the effective constants of W and Z bosons $\alpha_W(q^2)$ and $\alpha_Z(q^2)$ in the region $0 < q^2 \lesssim m_Z^2$ 'crawl' rather than run [80].

If we define the effective gauge coupling constant $g^2(q^2)$ in terms of the bare charge g_0^2 and the bare mass m_0, and sum up the geometric series with the self-energy $\Sigma(q^2)$ inserted in the gauge boson propagator, this gives the expression

$$g^2(q^2) = \frac{g_0^2}{1 + g_0^2 \frac{\Sigma(q^2)-\Sigma(m^2)}{q^2-m^2}}. \tag{C.1}$$

Here m is the physical mass, and $\Sigma(q^2)$ contains the contribution of fermions only, since loops with W, Z and H bosons do not contain large logarithms in the region $|q^2| \leqslant m_Z^2$.

The bare coupling constant in the difference $1/g^2(q^2) - 1/g^2(0)$ is eliminated, which gives a finite expression. The result is

$$1/\alpha_Z(q^2) - 1/\alpha_Z(0) = b_Z F(x), \qquad \text{where} \quad x = q^2/m_Z^2, \tag{C.2}$$

$$1/\alpha_W(q^2) - 1/\alpha_W(0) = b_W F(y), \qquad \text{where} \quad y = q^2/m_Z^2, \tag{C.3}$$

$$F(x) = \frac{x}{1-x} \log|x|. \tag{C.4}$$

If $x \gg 1$, equations (C.2) and (C.3) define the logarithmic running of charges owing to leptons and quarks, and b_Z and b_W represent the contribution of fermions to the first coefficient of the Gell-Mann–Low function:

$$b_Z = \frac{1}{48\pi}\left\{N_u 3\left[1+\left(1-\frac{8}{3}s^2\right)^2\right] + N_d 3\left[1+\left(-1+\frac{4}{3}s^2\right)^2\right] + N_l[2+(1+(1-4s^2)^2)]\right\}, \tag{C.5}$$

$$b_W = \frac{1}{16\pi}[6N_q + 2N_l],$$

where $N_{u,d,q,l}$ are the numbers of quarks and leptons with masses that are considerably lower than $\sqrt{q^2}$.

For $q^2 \lesssim m_Z^2$, the numerical values of the coefficients $b_{Z,W}$ are [80]:

$$b_Z \simeq 0.195 \qquad b_W \simeq 0.239.$$

The massive propagator $\frac{1}{q^2-m^2}$ in (C.1) greatly suppresses the running of $\alpha_W(q^2)$ and $\alpha_Z(q^2)$. Thus, according to (C.2) and (C.3), the constant $\alpha_Z(q^2)$ grows by 0.85% from $q^2 = 0$ to $q^2 = m_Z^2$,

$$\begin{aligned} &1/\alpha_Z(m_Z^2) = 22.905 \\ &1/\alpha_Z(m_Z^2) - 1/\alpha_Z(0) = -0.195, \end{aligned} \tag{C.6}$$

and the constant $\alpha_W(q^2)$ grows by 0.95%,

$$\begin{aligned} &1/\alpha_W(m_Z^2) = 28.74 \\ &1/\alpha_W(m_Z^2) - 1/\alpha_W(0) = -0.272, \end{aligned} \tag{C.7}$$

while the electromagnetic constant $\alpha(q^2)$ increases by 6.34%:

$$1/\alpha(m_Z^2) - 1/\alpha_W(0) = 128.90 - 137.04 = -8.14. \tag{C.8}$$

With the accuracy indicated above, we can thus assume

$$\begin{aligned} \alpha_Z(m_Z^2) &\simeq \alpha_Z(0) \\ \alpha_W(m_Z^2) &\simeq \alpha_W(0). \end{aligned} \tag{C.9}$$

At the same time, $\alpha(m_Z^2)$ differs greatly from $\alpha(0)$; therefore the latter has no connection to the electroweak physics but only to the purely electromagnetic physics.

Appendix D. General expressions for one-loop corrections to hadronless observables

The bare quantities are marked by the subscript '0'. In the electroweak theory, three bare charges e_0, f_0 and g_0 that describe the interactions of γ, Z and W are related by a single constraint:

$$(e_0/g_0)^2 + (g_0/f_0)^2 = 1. \tag{D.1}$$

The bare masses of the vector bosons are defined by the bare VEV of the higgs field η:

$$m_{Z0} = \tfrac{1}{2} f_0\eta, \qquad m_{W0} = \tfrac{1}{2} g_0\eta. \tag{D.2}$$

The fine structure constant $\alpha = e^2/4\pi$ is related to the bare charge e_0 by the formula

$$\alpha \equiv \alpha(q^2 = 0) = \frac{e_0^2}{4\pi}\left(1 - \Sigma'_\gamma(0) - 2\frac{s}{c}\frac{\Sigma_{\gamma Z}(0)}{m_Z^2}\right), \tag{D.3}$$

where $\Sigma'(0) = \lim_{q^2\to 0}\Sigma(q^2)/q^2$. In the Feynman gauge $\Sigma_{\gamma Z}(0) \approx -(\alpha/2\pi)(m_W^2/cs)(1/\epsilon)$, where the dimension of spacetime is $D = 4 - 2\varepsilon$. In the unitary gauge $\Sigma_{\gamma Z}(0) = 0$.

The simplest way to verify the presence of the term $2(s/c)\Sigma_{\gamma Z}(0)/m_Z^2$ is by considering the interaction of a photon with the right-handed electron e_R. Note that in this case there are no weak vertex corrections due to the W boson exchange. (Note also, that the left-handed neutrino remains neutral even when loop corrections are taken into account, since the diagram with the $\gamma - Z - \nu_L\bar{\nu}_L$ interaction is compensated for by the vertex diagram with the W-exchange.)

The relation between $\bar{\alpha} = \alpha(q^2 = m_Z^2)$ and α_0 has the following form:

$$\bar{\alpha} = \alpha_0\left[1 - \tilde{\Pi}_\gamma(m_Z^2) - \Sigma'_\gamma(0) + \tilde{\Sigma}'_\gamma(0) - 2\frac{s}{c}\Pi_{\gamma Z}(0)\right], \tag{D.4}$$

where $\tilde{\Pi}_\gamma(q^2) = \tilde{\Sigma}_\gamma(q^2)/m_Z^2$, $\Pi_{\gamma Z}(q^2) = \Sigma_{\gamma Z}(q^2)/m_Z^2$, while $\tilde{\Sigma}_\gamma$ mean that the contributions of the W boson and t quark are not accounted for. It is convenient to introduce in (D.4) explicit expression for $\delta\alpha_W + \delta\alpha_t$:

$$\bar{\alpha} = \alpha_0\left[1 - \Pi_\gamma(m_Z^2) - 2\frac{s}{c}\Pi_{\gamma Z}(0) - \delta\alpha_W - \delta\alpha_t\right], \tag{D.5}$$

where in accordance with equation (B.3)

$$\delta\alpha_W + \delta\alpha_t = \tilde{\Pi}_\gamma(m_Z^2) - \Pi_\gamma(m_Z^2) + \Sigma'_\gamma(0) - \tilde{\Sigma}'_\gamma(0). \tag{D.6}$$

In the case of 'new physics' one should add to equation (D.5) the term $\delta\alpha_{NP}$. Our first basic equation is equation (D.5).

The second basic equation is:

$$m_Z^2 = m_{Z0}^2[1 - \Pi_Z(m_Z^2)] = m_{W0}^2/c_0^2[1 - \Pi_Z(m_Z^2)]. \tag{D.7}$$

A similar equation holds for m_W^2:

$$m_W^2 = m_{W0}^2[1 - \Pi_W(m_W^2)], \tag{D.8}$$

where $\Pi_i(q^2) = \Sigma_i(q^2)/m_i^2$, $i = W, Z$.

Finally, the third basic equation is

$$G_\mu = \frac{g_0^2}{4\sqrt{2}m_{W0}^2}[1 + \Pi_W(0) + D], \tag{D.9}$$

where $\Pi_W(0) = \Sigma_W(0)/m_W^2$ comes from the propagator of W, while D is the contribution of the box and the vertex diagrams (minus the electromagnetic corrections to the four-fermion interaction) to the muon decay amplitude. According to Sirlin [81],

$$D = \frac{\bar{\alpha}}{4\pi s^2}\left(6 + \frac{7 - 4s^2}{2s^2}\log c^2 + 4\Delta_W\right), \tag{D.10}$$

where

$$\Delta_W \equiv \Delta(m_W) = \frac{2}{4 - D} + \log 4\pi - \gamma - \log(m_W^2/\mu^2). \tag{D.11}$$

Now we are able to express f_0 and g_0 in terms of $\bar{\alpha}$, G_μ, m_Z and the loop corrections. From (D.2), (D.7) and (D.9) we obtain:

$$f_0^2 = 4\sqrt{2}G_\mu m_Z^2[1 - \Pi_W(0) + \Pi_Z(m_Z^2) - D]. \tag{D.12}$$

From (D.1), (D.5) and (D.12) we get:

$$c_0 \equiv \frac{g_0}{f_0} = c\left[1 + \frac{s^2}{2(c^2 - s^2)} \times\left(-2\frac{s}{c}\Pi_{\gamma Z}(0) - \Pi_\gamma(m_Z^2) - \delta\alpha_W - \delta\alpha_t + \Pi_Z(m_Z^2) - \Pi_W(0) - D\right)\right]. \tag{D.13}$$

The next step is to express m_W/m_Z, g_A and g_V through c, s and loop corrections. Let us start with m_W/m_Z. From (D.8) and (D.7) we get:

$$m_W/m_Z = c_0[1 - \tfrac{1}{2}\Pi_W(m_W^2) + \tfrac{1}{2}\Pi_Z(m_Z^2)]. \tag{D.14}$$

Substituting c_0 given by (D.13) we obtain:

$$\frac{m_W}{m_Z} = c + \frac{cs^2}{2(c^2 - s^2)}\left(\frac{c^2}{s^2}[\Pi_Z(m_Z^2) - \Pi_W(m_W^2)] + \Pi_W(m_W^2) - \Pi_W(0) - \Pi_\gamma(m_Z^2) - 2\frac{s}{c}\Pi_{\gamma Z}(0) - D - \delta\alpha_W - \delta\alpha_t\right). \tag{D.15}$$

In order to obtain expression for g_A we should recall that it is proportional to f_0 and take into account the Z boson wavefunction renormalization and $Z\bar{l}l$ vertex loop correction:

$$g_A = -\tfrac{1}{2} - \tfrac{1}{4}[\Pi_Z(m_Z^2) - \Pi_W(0) - D - \Sigma_Z'(m_Z^2) - 8csF_A], \tag{D.16}$$

where F_A originates from the vertex correction.

The last quantity is the ratio g_V/g_A. One-loop corrections come from $s_0^2 \equiv 1 - c_0^2$ (equation (D.13)), from vector and axial $Z\bar{l}l$ vertices and from $Z \to \gamma$ transition which contributes to g_V only:

$$\frac{g_V}{g_A} = 1 - 4s^2 - \frac{4c^2s^2}{c^2 - s^2}\left[2\frac{s}{c}\Pi_{\gamma Z}(0) + \Pi_\gamma(m_Z^2) + \delta\alpha_W + \delta\alpha_t - \Pi_Z(m_Z^2) + \Pi_W(0) + D\right] - 4csF_V + 4csF_A(1 - 4s^2) - 4cs\Pi_{\gamma Z}(m_Z^2). \tag{D.17}$$

Formulae (D.15)–(D.17) are derived in this appendix according to the ‘five steps’ procedure described in section 4.6. They describe finite one-loop corrections to hadronless observables.

It is easy to evaluate the contribution of t quark to physical observables in the approximation $\sim\alpha_W m_t^2$. In this approximation $\Pi_W(m_W^2) = \Pi_W(0)$, $\Pi_Z(m_Z^2) = \Pi_Z(0)$, $\Pi_Z(0) - \Pi_W(0) = 3\bar{\alpha}/16\pi s^2c^2 t$ and from equations (D.15)–(D.17) we get:

$$\frac{m_W}{m_Z} \approx c + \frac{3\bar{\alpha}c}{32\pi(c^2-s^2)s^2}t, \tag{D.18}$$

$$g_A \approx -\frac{1}{2}\left(1 + \frac{3\bar{\alpha}}{32\pi s^2c^2}t\right), \tag{D.19}$$

$$\frac{g_V}{g_A} \approx 1 - 4s^2 + \frac{3\bar{\alpha}}{4\pi(c^2-s^2)}t. \tag{D.20}$$

The corrections proportional to m_t^2 were first pointed out by Veltman [37], who emphasized the appearance of such corrections for a large difference $m_t^2 - m_b^2$ which violates the isotopic symmetry. In this review the coefficients in front of the factors t in equations (D.18)–(D.20) are used as coefficients for normalized radiative corrections V_i (see sections 4.2 and 4.3).

Appendix E. Radiators R_{Aq} and R_{Vq}

For decays to light quarks $q = u, d, s$, we neglect the quark masses and take into account the gluon exchanges in the final state up to terms $\sim\alpha_s^3$ [82–85], and also one-photon exchange in the final state and the interference of the photon and the gluon exchanges [86]. These corrections are slightly different for the vector and the axial channels.

For decays to quarks we have

$$\Gamma_q = \Gamma(Z \to q\bar{q}) = 12[g_{Aq}^2 R_{Aq} + g_{Vq}^2 R_{Vq}]\Gamma_0 \tag{E.1}$$

where the factors $R_{A,V}$ are responsible for the interaction in the final state. According to [82–85]:

$$R_{Vq} = 1 + \frac{\hat{\alpha}_s}{\pi} + \frac{3}{4}Q_q^2\frac{\bar{\alpha}}{\pi} - \frac{1}{4}Q_q^2\frac{\bar{\alpha}}{\pi}\frac{\hat{\alpha}_s}{\pi} + \left[1.409 + (0.065 + 0.015\log t)\frac{1}{t}\right]\left(\frac{\hat{\alpha}_s}{\pi}\right)^2$$
$$-12.77\left(\frac{\hat{\alpha}_s}{\pi}\right)^3 + 12\frac{\hat{m}_q^2}{m_Z^2}\frac{\hat{\alpha}_s}{\pi}\delta_{vm} \tag{E.2}$$

$$R_{Aq} = R_{Vq} - (2T_{3q})\left[I_2(t)\left(\frac{\hat{\alpha}_s}{\pi}\right)^2 + I_3(t)\left(\frac{\hat{\alpha}_s}{\pi}\right)^3\right]$$
$$-12\frac{\hat{m}_q^2}{m_Z^2}\frac{\hat{\alpha}_s}{\pi}\delta_{vm} - 6\frac{\hat{m}_q^2}{m_Z^2}\delta_{am}^1 - 10\frac{\hat{m}_q^2}{m_t^2}\left(\frac{\hat{\alpha}_s}{\pi}\right)^2\delta_{am}^2, \tag{E.3}$$

where $\hat{m}_q$ is the running quark mass (see below),

$$\delta_{vm} = 1 + 8.7\left(\frac{\hat{\alpha}_s}{\pi}\right) + 45.15\left(\frac{\hat{\alpha}_s}{\pi}\right)^2, \tag{E.4}$$

$$\delta_{am}^1 = 1 + 3.67\left(\frac{\hat{\alpha}_s}{\pi}\right) + (11.29 - \log t)\left(\frac{\hat{\alpha}_s}{\pi}\right)^2, \tag{E.5}$$

$$\delta_{am}^2 = \frac{8}{81} + \frac{\log t}{54}, \tag{E.6}$$

$$I_2(t) = -3.083 - \log t + \frac{0.086}{t} + \frac{0.013}{t^2}, \tag{E.7}$$

$$I_3(t) = -15.988 - 3.722\log t + 1.917\log^2 t, \tag{E.8}$$

$$t = m_t^2/m_Z^2.$$

Terms of the order of $(\hat{\alpha}_s/\pi)^3$ caused by the diagrams with three gluons in intermediate state were calculated in [85]. For R_{Vq} they are numerically very small, $\sim 10^{-5}$; for this reason, we dropped them from formula (E.2).

For the $Z \to b\bar{b}$ decay, the b quark mass is not negligible; it reduces Γ_b by about 1 MeV ($\sim$0.5%). The gluon corrections result in a replacement of the pole mass $m_b \simeq 4.7$ GeV by the running mass at $q^2 = m_Z^2$: $m_b \to \hat{m}_b(m_Z)$. We express $\hat{m}_b(m_Z)$ in terms of m_b, $\hat{\alpha}_s(m_Z)$ and $\hat{\alpha}_s(m_b)$ using standard two-loop equations in the $\overline{MS}$ scheme (see [36]).

For the $Z \to c\bar{c}$ decay, the running mass $\hat{m}_c(m_Z)$ is of the order of 0.8 GeV and the corresponding contribution to Γ_c is of the order of 0.05 MeV. We have included this tiny term in the LEPTOP code, since it is taken into account in other codes (see, for example, [20]).

We need to remark in connection with Γ_c that the term $I_2(t)$, given by equation (E.7), contains interference terms $\sim(\hat{\alpha}_s/\pi)^2$. These terms are related to three types of final states: one quark pair, a quark pair and a gluon, two quark pairs. This last contribution comes to about 5% of I_2 and is below the currently achievable experimental accuracy. Nevertheless, in principle these terms require special consideration, especially if these quark pairs are of different flavours, for example, $b\bar{b}c\bar{c}$. Such mixed quark pairs must be discussed separately.

Note that $\hat{\alpha}_s$ stands for the strong interaction constant in the $\overline{MS}$ subtraction scheme, with $\mu^2 = m_Z^2$.

Appendix F. $\alpha_W^2 t^2$ corrections from reducible diagrams

In [33], when deriving equations for physical observables we systematically took into account corrections which contained first power of polarization operators and neglected terms $\sim(\Pi_{W,Z})^2$. This procedure was correct at one loop, but since $\Pi_{W,Z}$ contain terms of the order of $\alpha_W t$ we evidently lost $\alpha_W^2 t^2$ terms. To restore them let us repeat the procedure implemented in [33] this time taking squares of Π_W and Π_Z (reducible two-loop diagrams) into account.

Our starting point are three basic equations for quantities m_Z, G_μ and $\bar{\alpha} = \alpha(m_Z^2)$. Since terms $\sim\alpha_W t$ are absent in Π_γ, $\Pi_{\gamma Z}$ and D functions, we will not consider these functions in this section. Equation for m_Z is the same as in appendix D, equations (D.7), (D.2):

$$m_Z^2 = \tfrac{1}{4} f_0^2 \eta^2 [1 - \Pi_Z(m_Z^2)], \tag{F.1}$$

while for G_μ we have:

$$G_\mu = \frac{g_0^2}{\sqrt{2} g_0^2 \eta^2 [1 - \Pi_W(0)]} = \frac{1}{\sqrt{2}\eta^2 (1 - \Pi_W(0))}, \tag{F.2}$$

and we keep $\Pi_W(0)$ in the denominator to avoid losing the $\Pi_W^2(0)$ term (compare with equation (D.9)). From these two equations we get:

$$f_0^2 = 4\sqrt{2} G_\mu m_Z^2 \frac{1 - \Pi_W(0)}{1 - \Pi_Z(m_Z^2)} = 4\sqrt{2} G_\mu m_Z^2 \frac{1 - \Pi_W(0)}{1 - \Pi_Z(0)}, \tag{F.3}$$

where we use equality $\Pi_Z(m_Z^2) = \Pi_Z(0)$ which is valid for the leading term $\sim\alpha_W t$.

Finally, dividing the equation for the running electromagnetic coupling constant, which in our approximation is simply

$$e^2(m_Z^2) = 4\pi\bar{\alpha} = g_0^2 \left(1 - \frac{g_0^2}{f_0^2}\right) \tag{F.4}$$

by (F.3), we obtain:

$$\frac{g_0^2}{f_0^2}\left(1 - \frac{g_0^2}{f_0^2}\right) = \frac{\pi\bar{\alpha}}{\sqrt{2} G_\mu m_Z^2}[1 - \delta], \tag{F.5}$$

$$\delta \equiv 1 - \frac{1-\Pi_Z(0)}{1-\Pi_W(0)} = \frac{\Pi_Z(0)-\Pi_W(0)}{1-\Pi_W(0)}. \tag{F.6}$$

Considering δ as a small parameter and solving equation (F.5) perturbatively, we get:

$$\frac{g_0^2}{f_0^2} = c^2\left[1 + \frac{s^2}{c^2-s^2}\delta - \frac{c^2 s^4}{(c^2-s^2)^3}\delta^2\right], \tag{F.7}$$

where we keep terms linear and quadratic in δ. For definitions of c and s see equation (23).

Our next step should be the calculation of the δ^2 corrections to the functions V_i. But first let us discuss the expression for δ as given by equation (F.6) which contains the factor $1-\Pi_W(0)$ in the denominator. At one loop, corrections proportional to δ appear in physical observables. They should be carefully calculated in order not to induce extra $\alpha_W^2 t^2$ terms. Fortunately, this can be done straightforwardly, using the following chains of equalities:

$$\Pi_Z(0) \equiv \frac{4}{f_0^2\eta^2} f_0^2[\Sigma_Z(0)/f_0^2] = 4\sqrt{2}G_\mu(1-\Pi_W(0))[\Sigma_Z(0)/f_0^2], \tag{F.8}$$

$$\Pi_W(0) \equiv \frac{4}{g_0^2\eta^2} g_0^2[\Sigma_W(0)/g_0^2] = 4\sqrt{2}G_\mu(1-\Pi_W(0))[\Sigma_W(0)/g_0^2], \tag{F.9}$$

where expressions in square brackets contain self-energies *without* coupling constants (Σ_Z/f_0^2 and Σ_W/g_0^2, respectively) and equation (F.2) is used to express η through G_μ. Substituting equations (F.8) and (F.9) into (F.6) we get:

$$\delta = 4\sqrt{2}G_\mu[\Sigma_Z(0)/f_0^2 - \Sigma_W(0)/g_0^2] = 4\sqrt{2}G_\mu m_Z^2\frac{[\Sigma_Z(0)/f_0^2 - \Sigma_W(0)/g_0^2]}{m_Z^2}$$
$$= \frac{3\bar{\alpha}}{16\pi s^2 c^2}\left(\frac{m_t}{m_Z}\right)^2. \tag{F.10}$$

Now everything is prepared for the calculation of δ^2 corrections to physical observables. Let us start from the W boson mass. For the ratio of the squares of vector boson masses we have:

$$\frac{m_W^2}{m_Z^2} = \frac{g_0^2}{f_0^2}\frac{1-\Pi_W(m_W^2)}{1-\Pi_Z(m_Z^2)}. \tag{F.11}$$

Taking the ratio of bare coupling constants from equation (F.7) we get:

$$\frac{m_W}{m_Z} = c\sqrt{\frac{1-\Pi_W(0)}{1-\Pi_Z(0)}}\left[1 + \frac{s^2}{2(c^2-s^2)}\delta + \frac{s^6-5s^4c^2}{(c^2-s^2)^3}\frac{\delta^2}{8}\right]. \tag{F.12}$$

It is easy to see that:

$$\sqrt{\frac{1-\Pi_W(0)}{1-\Pi_Z(0)}} = \frac{1}{\sqrt{\frac{1-\Pi_Z(0)}{1-\Pi_W(0)}}} = \frac{1}{\sqrt{1-\delta}}. \tag{F.13}$$

The resulting formula for the correction to the ratio m_W/m_Z is presented in section 7.1.

The next step is the correction to the axial coupling of the Z boson to charged leptons. Axial coupling is proportional to f_0, and from equations (F.3) and (F.13) we immediately obtain:

$$f_0 \sim \frac{1}{\sqrt{1-\delta}} = 1 + \frac{1}{2}\delta + \frac{3}{8}\delta^2. \tag{F.14}$$

The final formula for the correction to g_{Al} is presented in section 7.1.

For the ratio of vector to axial constants in our approximation we have:

$$g_{Vl}/g_{Al} = 1 - 4s_0^2 = 1 - 4\left(1 - \frac{g_0^2}{f_0^2}\right). \tag{F.15}$$

The expression for the correction to g_{Vl}/g_{Al} through physical parameters is presented in section 7.1 as well.

Appendix G. Oblique corrections from new generations and SUSY

In this appendix we collect analytical formulae for different oblique corrections.

For the degenerate case the contribution of additional quark and lepton to $\delta^4 V_i$ are given by ([63]):

$$\begin{aligned}\delta^4 V_m = \tfrac{4}{9}N_c\{&[(1-l)F(l) - (1-l/c^2)F(l/c^2)] + 2s^2[(1-l/c^2)F(l/c^2)\\ &-(1+2l)F(l)] + 4s^4(Q_U^2+Q_D^2)[(1+2l)F(l) - \tfrac{1}{3}]\}\end{aligned} \tag{G.1}$$

$$\begin{aligned}\delta^4 V_A = \tfrac{4}{9}N_c\{&[1-l+(6l^2-3l)F(l)]\\ &+[4s^4(Q_U^2+Q_D^2) - 2s^2][2l+1-12l^2F(l)]\}/(1-4l),\end{aligned} \tag{G.2}$$

$$\delta^4 V_R = -\tfrac{4}{9}N_c\{3lF(l) - 4s^2c^2(Q_U^2+Q_D^2)[(1+2l)F(l) - \tfrac{1}{3}]\}, \tag{G.3}$$

where $N_c = 3$, $Q_U = \frac{2}{3}$, $Q_D = -\frac{1}{3}$ for quark doublet; $N_c = 1$, $Q_U = 0$, $Q_D = -1$ for lepton doublet;

$$\ell = m_Q^2/m_Z^2 \quad \text{for quarks}, \qquad \ell = m_L^2/m_Z^2 \quad \text{for leptons},$$

and the function $F(l)$ is defined in appendix B, equations (B8), (B.10).

For different up and down quark (and lepton) masses analytical expressions for $\delta^4 V_i$ are given by

$$\begin{aligned}\frac{1}{n}\delta^4 V_m = &\left(\frac{64}{27}s^4 - \frac{16}{9}s^2\right)\left[(1+2u)F(u) + (1+2d)F(d) - \frac{2}{3}\right]\\ &+\frac{8}{9}\left[(1-u)F(u) + (1-d)F(d) - \frac{2}{3}\right] + \frac{4}{3}\frac{s^2}{c^2}\left[u + d - \frac{2ud}{u-d}\log\frac{u}{d}\right]\\ &+\frac{8}{9}\left(1 - \frac{s^2}{c^2}\right)\left[\frac{u-d}{2}\log\frac{u}{d} + (u+d) + \left(c^2 - \frac{u+d}{2}\right)\frac{u+d}{u-d}\log\frac{u}{d} - \frac{4}{3}c^2\right.\\ &\left.-\left(2c^2 - u - d - \frac{(u-d)^2}{c^2}\right)F(m_W^2, m_U^2, m_D^2)\right];\end{aligned} \tag{G.4}$$

$$\begin{aligned}\frac{1}{n}\delta^4 V_A = \frac{4}{9}\Bigg\{&\left(\frac{16}{3}s^4 - 4s^2 - 1\right)[2uF(u) - (1+2u)F'(u) + 2dF(d) - (1+2d)F'(d)]\\ &+3\left[u + d - \frac{2ud}{u-d}\log\frac{u}{d} - F'(u) - F'(d)\right]\Bigg\};\end{aligned} \tag{G.5}$$

$$\begin{aligned}\frac{1}{n}\delta^4 V_R = -\frac{8}{3}&\left[uF(u) + dF(d) + \frac{ud}{u-d}\log\frac{u}{d} - \frac{u+d}{2}\right]\\ &+\frac{64}{27}s^2c^2\left[(1+2u)F(u) + (1+2d)F(d) - \frac{2}{3}\right],\end{aligned} \tag{G.6}$$

where n is the number of generations and $m_N = m_U$, $m_E = m_D$, $u = m_U^2/m_Z^2$, $d = m_D^2/m_Z^2$; $F'(u) = -u(\mathrm{d}/\mathrm{d}u)F(u)$ and $F(s, m_1^2, m_2^2)$ is defined in appendix B, equation (B.6).

The formulae that describe the enhanced SUSY corrections to the functions V_i have the following form [72]:

$$\delta^{LR}_{SUSY} V_A = \frac{1}{m_Z^2}[c_u^2 g(m_1, m_{\tilde{b}}) + s_u^2 g(m_2, m_{\tilde{b}}) - c_u^2 s_u^2 g(m_1, m_2)], \tag{G.7}$$

$$\delta^{LR}_{SUSY} V_R = \delta^{LR}_{SUSY} V_A + \frac{1}{3} Y_L \left[c_u^2 \log\left(\frac{m_1^2}{m_{\tilde{b}}^2}\right) + s_u^2 \log\left(\frac{m_2^2}{m_{\tilde{b}}^2}\right)\right] - \frac{1}{3} c_u^2 s_u^2 h(m_1, m_2), \tag{G.8}$$

$$\delta^{LR}_{SUSY} V_m = \delta^{LR}_{SUSY} V_A + \frac{2}{3} Y_L s^2 \left[c_u^2 \log\left(\frac{m_1^2}{m_{\tilde{b}}^2}\right) + s_u^2 \log\left(\frac{m_2^2}{m_{\tilde{b}}^2}\right)\right]$$
$$+ \frac{c^2 - s^2}{3}[c_u^2 h(m_1, m_{\tilde{b}}) + s_u^2 h(m_2, m_{\tilde{b}})] - \frac{c_u^2 s_u^2}{3} h(m_1, m_2), \tag{G.9}$$

where

$$g(m_1, m_2) = m_1^2 + m_2^2 - 2\frac{m_1^2 m_2^2}{m_1^2 - m_2^2} \log\left(\frac{m_1^2}{m_2^2}\right), \tag{G.10}$$

$$h(m_1, m_2) = -\frac{5}{3} + \frac{4m_1^2 m_2^2}{(m_1^2 - m_2^2)^2} +$$
$$\frac{(m_1^2 + m_2^2)(m_1^4 - 4m_1^2 m_2^2 + m_2^4)}{(m_1^2 - m_2^2)^3} \log\left(\frac{m_1^2}{m_2^2}\right), \tag{G.11}$$

and $Y_L = Q_t + Q_b = \frac{1}{3}$ is the hypercharge of the left doublet.

Appendix H. Other parametrizations of radiative corrections

Here we present formulae which connect our functions V_i with two other sets of parameters widely used in the literature to describe electroweak radiative corrections. All formulae of this appendix are valid at one-electroweak-loop approximation.

A set of three parameters $\varepsilon_1, \varepsilon_2, \varepsilon_3$ has been suggested by Altarelli, Barbieri and Jadach [87] for the phenomenological analysis of new physics:

$$\varepsilon_1 = \Delta\rho, \tag{H.1}$$

$$\varepsilon_2 = c^2 \Delta\rho + \frac{s^2}{c^2 - s^2} \Delta r_W - 2s^2 \Delta k', \tag{H.2}$$

$$\varepsilon_3 = c^2 \Delta\rho + (c^2 - s^2)\Delta k', \tag{H.3}$$

where $\Delta\rho$ describes the correction to g_A, $\Delta k'$ to g_V and Δr_W to m_W/m_Z:

$$g_A = -\tfrac{1}{2}(1 + \tfrac{1}{2}\Delta\rho), \tag{H.4}$$

$$g_V/g_A = 1 - 4s^2(1 + \Delta k'), \tag{H.5}$$

$$m_W/m_Z = c[1 - s^2 \Delta r_W / 2(c^2 - s^2)]. \tag{H.6}$$

By comparing these definitions with the definitions of V_A, V_R and V_m we obtain:

$$\Delta\rho = \frac{3\bar{\alpha}}{16\pi} \frac{V_A}{s^2 c^2}, \tag{H.7}$$

$$\Delta k' = -\frac{3\bar{\alpha}}{16\pi} \frac{V_R}{(c^2 - s^2)s^2}, \tag{H.8}$$

$$\Delta r_W = -\frac{3\bar{\alpha}}{16\pi} \frac{V_m}{s^4}. \tag{H.9}$$

Hence:

$$\varepsilon_1 = \frac{3\bar{\alpha}}{16\pi s^2 c^2} V_A, \tag{H.10}$$

$$\varepsilon_2 = \frac{3\bar{\alpha}}{16\pi (c^2 - s^2) s^2} [(V_A - V_m) - 2s^2 (V_A - V_R)], \tag{H.11}$$

$$\varepsilon_3 = \frac{3\bar{\alpha}}{16\pi s^2} (V_A - V_R). \tag{H.12}$$

As is evident from the last two formulae, the virtue of ε_2 and ε_3 is that they do not contain the term t. So, at the time when t quark mass was not measured at Tevatron the corresponding uncertainties in ε_2 and ε_3 were diminished.

Another set of parameters, S, T, U was introduced a few years earlier by Peskin and Takeuchi [88]. These parameters were proposed to describe only the so-called oblique corrections due to the physics beyond the SM. Using the definitions of S, T, U from [88] and designating new physics contributions to ε_i as $\delta\varepsilon_i$ we obtain:

$$\delta\varepsilon_1 = \bar{\alpha} T, \tag{H.13}$$

$$\delta\varepsilon_2 = -\bar{\alpha} U / 4s^2, \tag{H.14}$$

$$\delta\varepsilon_3 = \bar{\alpha} S / 4s^2. \tag{H.15}$$

From (H.10)–(H.12) we get:

$$T = \frac{3}{16\pi s^2 c^2} \delta V_A, \tag{H.16}$$

$$U = -\frac{3}{4\pi (c^2 - s^2)} [(\delta V_A - \delta V_m) - 2s^2 (\delta V_A - \delta V_R)], \tag{H.17}$$

$$S = \frac{3}{4\pi} (\delta V_A - \delta V_R), \tag{H.18}$$

where δV_i are new physics contributions to V_i.

According to [88]

$$S = 16\pi [\Sigma'_{33}(0) - \Sigma'_{3Q}(0)] = 16\pi [\Sigma'_A(0) - \Sigma'_V(0)], \tag{H.19}$$

$$T = \frac{4\pi}{s^2 m_W^2} [\Sigma_{11}(0) - \Sigma_{33}(0)], \tag{H.20}$$

$$U = 16\pi [\Sigma'_{11}(0) - \Sigma'_3(0)], \tag{H.21}$$

where $\Sigma'(0) = \mathrm{d}\Sigma(q^2)/\mathrm{d}q^2|_{q^2=0}$ and Σ are defined by the corresponding currents (isotopic, 1 and 3, and electromagnetic, Q, vector, V, and axial, A) with coupling constants being extracted. Thus S characterizes the degree of chiral symmetry breaking, and T and U that of isotopic symmetry. Note that in equations (H.19)–(H.21) only the contribution of new physics should be considered. Since new particles should be heavy it is reasonable to take into account only values of self-energies at $q^2 = 0$ and their first derivatives (higher derivatives are power suppressed) [88]. Altogether we have eight parameters ($\Sigma_{WW}(0)$, $\Sigma_{ZZ}(0)$, $\Sigma_{\gamma Z}(0)$, $\Sigma_{\gamma\gamma}(0)$, $\Sigma'_{WW}(0)$, $\Sigma'_{ZZ}(0)$, $\Sigma'_{Z\gamma}(0)$, $\Sigma'_{\gamma\gamma}(0)$), two of which are equal to zero ($\Sigma_{\gamma\gamma}(0)$ and $\Sigma_{\gamma Z}(0)$), while three combinations can be absorbed in the definition of α, G_μ and m_Z. The remaining three combinations enter S, T and U (or $\delta\varepsilon_i$, $i = 1, 2, 3$).

References

[1] Arnison G *et al* (UA1 Collaboration) 1983 *Phys. Lett.* B **126** 398
Bagnaia P *et al* (UA2 Collaboration) 1983 *Phys. Lett.* B **129** 310
Arnison G *et al* (UA1 Collaboration) 1983 *Phys. Lett.* B **122** 103

Banner M *et al* (UA2 Collaboration) 1983 *Phys. Lett.* B **122** 476
[2] Van der Meer S 1985 *Rep. Mod. Phys.* **57** 689
Rubbia C 1985 *Rep. Mod. Phys.* **57** 699
[3] Glashow S L 1961 *Nucl. Phys.* **22** 579
Weinberg S 1967 *Phys. Rev. Lett.* **19** 1264
Salam A 1968 *Elementary Particle Theory* ed N Svartholm (Stockholm: Almquist and Wiksells)
[4] Glashow S L 1980 *Rev. Mod. Phys.* **52** 539
Weinberg S 1980 *Rev. Mod. Phys.* **52** 515
Salam A 1980 *Rev. Mod. Phys.* **52** 525
[5] Peskin M E and Shroeder D V 1995 *An Introduction to Quantum Field Theory* (Addison-Wesely)
Weinberg S 1995 *The Quantum Theory of Fields* (Cambridge: Cambridge University Press) p 1996
[6] Fermi E 1934 *Z. Phys.* **88** 161
[7] Feynman R P and Gell-Mann M 1958 *Phys. Rev.* **109** 193
Marshak R E and Sudarshan E C G 1958 *Phys. Rev.* **109** 1860
Sakurai J J 1958 *Nuovo Cimento* **7** 649
[8] Klein O 1938 On the theory of charged fields *New Theories in Physics* (Warsaw) p 66 (reprinted in 1991 *Oskar Klein Memorial lectures* vol 1, ed G Ekspong (Singapore: World Scientific))
[9] Yang C N and Mills R L 1954 *Phys. Rev.* **96** 191
[10] Higgs P W 1964 *Phys. Rev. Lett.* **12** 508
Higgs P W 1966 *Phys. Rev.* **145** 1156
Englert F and Brout R 1964 *Phys. Rev. Lett.* **13** 321
Guralnik G S, Hagen C R and Kibble T W 1964 *Phys. Rev. Lett.* **13** 585
Kibble T W 1967 *Phys. Rev.* **155** 1554
[11] 't Hooft G 1971 *Nucl. Phys.* B **33** 173
't Hooft G 1971 *Nucl. Phys.* B **35** 167
[12] Hasert F *et al* 1974 *Nucl. Phys.* B **73** 1
Benvenuti A *et al* 1974 *Phys. Rev. Lett.* **32** 800
[13] Cnops A M *et al* 1978 *Phys. Rev. Lett.* **41** 357
Bergsma F *et al* 1984 *Phys. Lett.* B **147** 481
[14] Prescott C Y *et al* 1978 *Phys. Lett.* B **77** 347
Prescott C Y *et al* 1979 *Phys. Lett.* B **84** 524
[15] Barkov L M and Zolotorev M S 1978 *Pis'ma v ZhETF* **26** 544
Barkov L M and Zolotorev M S 1979 *Phys. Lett.* B **85** 308
Khriplovich I B 1991 *Parity Nonconservation in Atomic Phenomena* (Harwood Academic Publishers)
Blundell S A, Johnson W R and Sapirstein J 1990 *Phys. Rev. Lett.* **65** 1411
Bouchiat M A and Pottier L 1986 *Science* **234** 1203
Rosner J and Marciano W 1990 *Phys. Rev. Lett.* **65** 2963
[16] Bardin D Yu and Dokuchaeva V A 1984 *Nucl. Phys.* B **246** 221
Sarantakos S, Sirlin A and Marciano W J 1983 *Nucl. Phys.* B **227** 84
Stuart R G 1987 *Z. Phys.* C **34** 445
[17] Sirlin A and Marciano W J 1980 *Phys. Rev.* D **22** 2695
Sirlin A and Marciano W J 1981 *Nucl. Phys.* B **189** 442
Wheater J F and Llewellynn-Smith C H 1982 *Nucl. Phys.* B **208** 27
Wheater J F and Llewellynn-Smith C H 1983 *Nucl. Phys.* B **226** 547
Llewellynn-Smith C H 1983 *Nucl. Phys.* B **228** 205
Bardin D Yu and Dokuchaeva V A 1986 *Preprint JINR* E2-86-260 (Dubna) unpublished
[18] Ellis J and Peccei R (eds) 1986 *Physics at LEP* CERN 86-02
[19] Altarelli G, Kleiss R and Verzegnassi C (eds) 1989 *Physics at LEP-I* CERN 89-08
[20] Bardin D, Hollik W and Passarino G (eds) 1995 *Reports of the Working Group on Precision Calculations for the Z resonance* CERN 95-03
[21] Bardin D and Passarino G 1999 *The Standard Model in the Making: Precision Study of Electroweak Interactions* (Oxford: Oxford University Press) to be published
[22] *LEP EWWG and SLD HFEWG* CERN-EP/99-15 8 February 1999
[23] Erler J and Langacker P 1998 *Preprint* UPR-0816-T, hep-ph/9809352
[24] Vilain P *et al* (CHARM II Collaboration) 1994 *Phys. Lett.* B **335** 246
[25] Dorenbosch J *et al* (CHARM Collaboration) 1986 *Phys. Lett.* B **180** 303
Allen R C *et al* (LAMPF Collaboration) 1993 *Phys. Rev.* D **47** 11
[26] Novikov V A, Okun L B and Vysotsky M I 1993 *Phys. Lett.* B **298** 453

Vilain P *et al* (CHARM II Collaboration) 1994 *Phys. Lett.* B **320** 203

[27] 1998 Review of particle physics *Eur. Phys. J.* C **3**

[28] Steinhauser M 1998 *Phys. Lett.* B **429** 158

[29] Eidelman S and Jegerlehner F 1995 *Z. Phys.* C **67** 585

[30] Davier M and Höcker A 1998 *Phys. Lett.* B **419** 419

[31] Berman S M and Sirlin A 1962 *Ann. Phys., NY* **20** 20
Sirlin A 1978 *Rev. Mod. Phys.* **50** 573

[32] Lynn B W and Peskin M E 1985 *Report* SLAC-PUB-3724 unpublished
Lynn B W, Peskin M E and Stuart R G 1986 *Physics at LEP* CERN 86-02 (CERN, Geneva)

[33] Novikov V, Okun L and Vysotsky M 1993 *Nucl. Phys.* B **397** 35

[34] Willenbrock S and Valencia G 1991 *Phys. Lett.* B **259** 573
Leike A, Riemann T and Rose J 1991 *Phys. Lett.* B **273** 513
Adriani O *et al* (L3 Collaboration) 1993 *Phys. Lett.* B **315** 494
Veltman H 1994 *Z. Phys.* C **62** 35

[35] Cabibbo N 1963 *Phys. Rev. Lett.* **10** 531
Kobayashi M and Maskawa T 1973 *Prog. Theor. Phys.* **49** 652

[36] Gray N *et al* 1990 *Z. Phys.* C **48** 673
Surguladze L R 1994 University of Oregon report No. OITS543 *Preprint* hep-ph/9405325

[37] Veltman M 1977 *Nucl. Phys.* B **123** 89
Veltman M 1977 *Acta Phys. Pol.* B **8** 475

[38] Abe F *et al* (CDF Collaboration)1995 *Phys. Rev. Lett.* **74** 2626

[39] Abachi S *et al* (D0 Collaboration) 1995 *Phys. Rev. Lett.* **74** 2632

[40] Politzer H D 1973 *Phys. Rev. Lett.* **30** 1346
Gross D J and Wilczek F 1973 *Phys. Rev. Lett.* **30** 1343
Gross D J and Wilczek F 1973 *Phys. Rev.* D **8** 3633

[41] Vysotsky M I, Novikov V A, Okun L B and Rozanov A N 1996 *Usp. Fiz. Nauk* **39** 503

[42] Novikov V, Okun L, Rozanov A and Vysotsky M 1995 *Preprint* ITEP 19-95
Novikov V, Okun L, Rozanov A and Vysotsky M 1995 *Preprint* CPPM-1-95
Novikov V, Okun L, Rozanov A and Vysotsky M http://cppm.in2p3.fr./leptop/intro_leptop.html

[43] Djouadi A and Verzegnassi C 1987 *Phys. Lett.* B **195** 265
Kniehl B A 1990 *Nucl. Phys.* B **347** 86
Halzen F and Kniehl B A 1991 *Nucl. Phys.* B **353** 517

[44] Novikov V, Okun L, Shifman M, Vainshtein A, Voloshin M and Zakharov V 1978 *Phys. Rep.* C **41** 1

[45] Chetyrkin K G, Kühn J H and Steinhauser M 1995 *Phys. Lett.* B **351** 331
Chetyrkin K G, Kühn J H and Steinhauser M 1995 *Phys. Rev. Lett.* **75** 3394
Avdeev L, Fleisher J, Mikhailov S and Tarasov O 1994 *Phys. Lett.* B **336** 560
Avdeev L, Fleisher J, Mikhailov S and Tarasov O 1995 *Phys. Lett.* B **349** 597

[46] Czarnecki A and Kühn J H 1996 *Phys. Rev. Lett.* **77** 3955

[47] Akhundov A A, Bardin D Yu and Riemann T 1986 *Nucl. Phys.* B **276** 1
Bernabeu J, Pich A and Santamaria A 1986 *Phys. Lett.* B **200** 569
Beenaker W and Hollik W 1988 *Z. Phys.* C **40** 141

[48] Fleisher J, Jegerlehner F, Raczka P and Tarasov O V 1992 *Phys. Lett.* B **293** 437
Buchalla C and Buras A 1990 *Nucl. Phys.* B **398** 285
Degrassi G 1993 *Nucl. Phys.* B **407** 271
Chetyrkin K G, Kwiatkowski A and Steinhauser M 1993 *Mod. Phys. Lett.* A **8** 2785

[49] Harlander R, Seidensticker T and Steinhauser M 1998 *Phys. Lett.* B **426** 125

[50] Bardin D *et al* 1991 Program `ZFITTER` 4.9 *Nucl. Phys.* B **351** 1
Bardin D *et al* 1989 *Z. Phys.* C **44** 493
Bardin D *et al* 1991 *Phys. Lett.* B **255** 290
Bardin D *et al* 1992 *Preprint* CERN-TH 6443-92

[51] Ellis J and Fogli G 1988 *Phys. Lett.* B **213** 189
Ellis J and Fogli G 1988 *Phys. Lett.* B **213** 526
Ellis J and Fogli G 1989 *Phys. Lett.* B **232** 139
Ellis J and Fogli G 1990 *Phys. Lett.* B **249** 543

[52] Hollik W 1990 *Fortschr. Phys.* **38** 3
Hollik W 1990 *Fortschr. Phys.* **38** 165
Consoli M, Hollik W and Jegerlehner F 1989 Proceedings of the workshop on Z physics at LEP-I *CERN Report* 89-08 vol 1, p 7

Burgers G, Jegerlehner F, Kniehl B and Kühn J H 1989 *CERN Report* 89-08 vol 1, p 55
[53] Montagna G, Nicrosini O, Passarino G, Piccinini F and Pittau R 1993 *Nucl. Phys.* B **401** 3
Montagna G, Nicrosini O, Passarino G, Piccinini F and Pittau R 1993 Program TOPAZO *Comput. Phys. Commun.* **76** 328
[54] Erler J and Langacker P 1998 *Eur. J. Phys.* **3** 90
[55] Novikov V, Okun L, Rozanov A and Vysotsky M 1995 *Preprint* ITEP 19-95, CPPM-1-95
Novikov V, Okun L, Rozanov A and Vysotsky M http://cppm.in2p3.fr./leptop/intro_leptop.html
[56] Barbieri R, Beccaria M, Ciafaloni P, Curci G and Vicere A 1992 *Phys. Lett.* B **288** 95
Barbieri R, Beccaria M, Ciafaloni P, Curci G and Vicere A 1993 *Nucl. Phys.* B **409** 105
[57] Fleischer J, Tarasov O V and Jegerlehner F 1993 *Preprint* BI-TP-93/24 and PSI-PR-93-14
Fleischer J, Tarasov O V and Jegerlehner F 1993 *Phys. Lett.* B **319** 249
[58] Degrassi G, Gambino P and Vicini A 1996 *Phys. Lett.* B **383** 219
[59] Degrassi G, Gambino P and Sirlin A 1997 *Phys. Lett.* B **394** 188
[60] Degrassi G, Gambino P, Passera M and Sirlin A 1998 *Phys. Lett.* B **418** 209
[61] Chanowitz M 1998 *Phys. Rev. Lett.*
Chanowitz M 1998 *Preprint* LBNL-42103; hep-ph/9807452
[62] Inami T, Kawakami T and Lim C S 1995 *Mod. Phys. Lett.* A **10** 1471
[63] Novikov V, Okun L, Rozanov A, Vysotsky M and Yurov V 1995 *Mod. Phys. Lett.* A **10** 1915
[64] Masiero A, Feruglio F, Rigolin S and Strocchi R 1995 *Phys. Lett.* B **355** 329
[65] Nilles H P 1984 *Phys. Rep.* **110** 1
Haber H E and Kane G L 1985 *Phys. Rep.* **117** 75
Peskin M 1997 Proc. 1996 European School of High-Energy Physics (Carry-le-Rouet, France) ed N Ellis and M Neubert *CERN Report* 97-03 p 49
(Peskin M 1997 *Preprint* hep-ph/9705479)
Ellis J 1998 Proc. 1998 European School of High-Energy Physics (St. Andrews, Scotland) *Preprint* CERN-TH/98-329, hep-ph/9812235
[66] Chankowski P *et al* 1994 *Nucl. Phys.* B **417** 101
[67] Garcia D, Jimenez R J and Sola J 1995 *Phys. Lett.* B **347** 309
Garcia D, Jimenez R J and Sola J 1995 *Phys. Lett.* B **347** 321
[68] de Boer W *et al* 1997 *Z. Phys.* C **75** 627
[69] Erler J and Pierce D M 1998 *Nucl. Phys.* B **526** 53
[70] Alvarez-Gaume L, Polchinski J and Wise M 1983 *Nucl. Phys.* B **211** 495
Barbieri R and Maiani L 1983 *Nucl. Phys.* B **224** 32
[71] Boulware M and Finnell D 1991 *Phys. Rev.* D **44** 2054
[72] Gaidaenko I V, Novikov A V, Novikov V A, Rozanov A N and Vysotsky M I 1998 *JETP Lett.* **67** 761
Gaidaenko I V, Novikov A V, Novikov V A, Rozanov A N, and Vysotsky M I 1998 *Preprint* hep-ph/9812346
[73] Hagiwara K and Murayama H 1990 *Phys. Lett.* B **246** 533
[74] Lammel S 1998 *FERMILAB-CONF-98-055-E*
[75] Chankowski P 1997 *Proc. Quantum effects in the MSSM, (Barcelona)* p 87
Chankowski P 1997 *Preprint* IFT/97-18, hep-ph/9711470
[76] Haber H E and Hempfling R 1991 *Phys. Rev. Lett.* **66** 1815
Okada Y, Yamaguchi M and Yanagida T 1991 *Prog. Theor. Phys.* **85** 1
Ellis J, Ridolfi G and Zwirner F 1991 *Phys. Lett.* B **257** 83
[77] Landau L D, Abrikosov A A and Khalatnikov I M 1954 *Dokl. Akad. Nauk SSSR* **95** 497
Landau L D, Abrikosov A A and Khalatnikov I M 1954 *Dokl. Akad. Nauk SSSR* **95** 1177
Gell-Mann M and Low F 1954 *Phys. Rev.* **95** 1300
[78] Altarelli G, Kleiss R and Verzegnassi C (eds) 1989 *Physics at LEP-I* vol 1(CERN 89-08, Geneva) p 7 (Conveners: M Consoli and W Hollik; Working Group: F Jegerlehner)
[79] Nevzorov R and Novikov A 1996 *Yad. Fiz.* **59** 540
[80] Novikov V A, Okun L B and Vysotsky M I 1994 *Mod. Phys. Lett.* A **9** 1489
[81] Sirlin A 1980 *Phys. Rev.* D **22** 971
[82] Gorishny S G, Kataev A L and Larin S A 1991 *Phys. Lett.* B **259** 144
Surguladze L R and Samuel M A 1991 *Phys. Rev. Lett.* **66** 560
[83] Chetyrkin K G and Kühn J H 1990 *Phys. Lett.* B **248** 359
Chetyrkin K G, Kühn J H and Kwiatkowski A 1992 *Phys. Lett.* **282** 221
[84] Chetyrkin K G and Kwiatkowski A 1993 *Phys. Lett.* B **305** 288
Chetyrkin K G and Kwiatkowski A 1993 *Phys. Lett.* B **319** 307
Chetyrkin K G 1993 *Phys. Lett.* B **307** 169

[85] Larin S A, van Ritberger T and Vermaseren J A M 1994 *Phys. Lett.* B **320** 159
Chetyrkin K G and Tarasov O V 1994 *Phys. Lett.* B **327** 114
[86] Kataev A L 1992 *Phys. Lett.* **287** 209
[87] Altarelli G and Barbieri R 1991 *Phys. Lett.* B **253** 161
Altarelli G, Barbieri R and Jadach S 1992 *Nucl. Phys.* B **369** 3
[88] Peskin M and Takeuchi T 1990 *Phys. Rev. Lett.* **65** 964
Peskin M and Takeuchi T 1992 *Phys. Rev.* D **46** 381
[89] ALEPH, DELPHI, L3 and OPAL Collaborations 1999 Limits on Higgs boson masses from combining the data of the four LEP experiments at $\sqrt{s} < 183$ GeV *Preprint* CERN-EP (in preparation)

Author queries

Note 1 (Page 3):
sense as intended?

Note 2 (Page 10):
ok to set above fractions as case fractions?

Note 3 (Page 13):
no table caption ok?

Note 4 (Page 16):
no table caption ok?

Note 5 (Page 16):
no table caption ok?

Note 6 (Page 28):
sense as intended?

Note 7 (Page 29):
please clarify?

Note 8 (Page 56):
page number ok?

Note 9 (Page 56):
please provide update?

Note 10 (Page 56):
please provide update?

Note 11 (Page 56):
please provide update?

Note 12 (Page 57):
please provide update?

Note 13 (Page 57):
please provide update?

Note 14 (Page 57):
please provide update?

Note 15 (Page 57):
please provide update?

Note 16 (Page 58):
please provide update?

Note 17 (Page 58):
please provide update?

Note 18 (Page 58):
please provide details?

Note 19 (Page 58):
please provide update?

Note 20 (Page 58):
please provide update?

Note 21 (Page 58):
please provide update?

Note 22 (Page 58):
please provide update?

Note 23 (Page 59):
please provide update?