Electronic Integration and Differentiation

— Version 1.0, formulas only (and probably some typos) —

Integrator (RC)

$$U_{\mathrm{R}} = I \cdot R$$

$$U_1 = U_{\mathrm{C}} = \frac{Q}{C} = \frac{\int dt\, I}{C}$$

$$I = \frac{dQ}{dt} \Leftrightarrow \int dt I = Q$$

$$U_1 = \frac{1}{RC} \int dt\, (U_0 - U_1)$$

$$\frac{d}{dt} U_1 + \frac{1}{RC} U_1 = \frac{1}{RC} U_0$$

From this equation, we derive two statements

A) In the stable situation, all functionals $\frac{d}{dt} = 0$. Hence, $U_1 = U_0$. (And therefore $\frac{d}{dt} U_1 = \frac{d}{dt} U_0$.)

B) The dynamic (interesting) case is $\frac{d}{dt} \gg \frac{1}{RC}$.

We obtain in that limit then

$$U_1 \approx \frac{1}{RC} \int dt\, U_0$$

The output voltage equals the integral of the input voltage.

Differentiator (CR)

$$U_1 = U_{\mathrm{R}} = I \cdot R = \frac{dQ}{dt} R = R \cdot C \cdot \frac{dU_{\mathrm{C}}}{dt} = R \cdot C \cdot \frac{d}{dt} (U_0 - U_1)$$

$$Q = C \cdot U_{\mathrm{C}}$$

$$U_1 = \frac{d}{dt} (R \cdot C \cdot U_1) = R \cdot C \frac{d}{dt} U_0$$

$$\frac{1}{RC} U_1 + \frac{d}{dt} U_1 = \frac{d}{dt} U_0$$

Until this equation, everything is exact again. We derive two statements again:

A) The stationary case is reached for $\frac{d}{dt} = 0 \Rightarrow U_1 = 0$.

B) We are interested in the dynamic case, which is characterized by $\frac{d}{dt} \ll \frac{1}{RC}$.

Thus we obtain the approximation

$$\frac{1}{RC} U_1 \approx \frac{d}{dt} U0,$$

which is the description of the situation, where the output represents the first derivative of the input signal.

Closing Remarks

. . . are due.